7. [13 points] The population of fish (in thousands) in a lake t years after 2010 is given by the function

$$F(t) = \frac{220}{1 + 2(1.35)^{-t}}.$$

a. [3 points] Find the value and give a practical interpretation of the vertical intercept of the function $F(t)$.

Solution: Vertical intercept= $F(0) = \dfrac{220}{1 + 2(1.35)^0} = \dfrac{220}{3} \approx 73.33$

Interpretation: There were $\frac{220}{3}$ thousand fish in the lake in 2010.

b. [4 points] When is the population in the lake equal to 150 thousand fish? Your answer must be found algebraically, written in exact form or rounded to the nearest 0.01.

Solution:

$$\begin{aligned}
\frac{220}{1 + 2(1.35)^{-t}} &= 150 \\
220 &= 150(1 + 2(1.35)^{-t}) \\
220 &= 150 + 300(1.35)^{-t} \\
300(1.35)^{-t} &= 70 \\
(1.35)^{-t} &= \frac{7}{30} \\
-t\ln(1.35) &= \ln\left(\frac{7}{30}\right) \\
t &= \frac{\ln\left(\frac{7}{30}\right)}{-\ln(1.35)} \approx 4.85 \text{ years after 2010.}
\end{aligned}$$

This problem continues on the next page.

The statement of the problem is included here for your convenience.

The population of fish (in thousands) in a lake t years after 2010 is given by the function

$$F(t) = \frac{220}{1 + 2(1.35)^{-t}}.$$

c. [3 points] Consider the graph of $y = F(t)$ for $-\infty < t < \infty$. Find the equation(s) of the horizontal asymptote(s) of the graph. If the graph has no horizontal asymptotes write "None".

Solution: The graph has horizontal asymptotes at $y = 0$ and $y = 220$.

d. [3 points] Find the average rate of change of $F(t)$ for $-1 \le t \le 5$. Include units.

Solution:
Average rate of change of $F(t)$ for $-1 \le t \le 5 = \frac{F(5) - F(-1)}{6} \approx \frac{152.14 - 59.46}{6}$

≈ 15.44 thousand fish per year.