9. [14 points]

a. [7 points] A mass is attached to the top of a ceiling by a spring. The height of the mass above the ground oscillates from a minimum of 1.2 meters to a maximum of 2.5 meters. Let $f(t)$ be the height of the mass above the ground, in meters, at time t measured in seconds. Some of the values of the function $f(t)$ are shown below

t	0	1	2	3	4
$f(t)$	1.65	2.38	2.38	1.65	1.2

Note: All the values in the table are rounded to the nearest 0.01.
Suppose $f(t)$ is a sinusoidal function.

i) Find the period, amplitude and midline of $y = f(t)$.

Solution: Period= 5 Amplitude=0.65 Midline: $y = 1.85$

ii) Find a formula for $f(t)$.

$$f(t) = 1.85 + 0.65\cos\left(\frac{2\pi}{5}(t - 1.5)\right)$$

b. [7 points] Find all solutions to $4 - 5\sin\left(\frac{\pi}{2}x - \frac{\pi}{6}\right) = 2$ for $0 \le x \le 5$. Your answers must be found algebraically and in **exact** form.

Solution:

$$\begin{aligned} 4 - 5\sin\left(\frac{\pi}{2}x - \frac{\pi}{6}\right) &= 2 \\ 5\sin\left(\frac{\pi}{2}x - \frac{\pi}{6}\right) &= -2 \\ \sin\left(\frac{\pi}{2}x - \frac{\pi}{6}\right) &= 0.4 \\ \frac{\pi}{2}x - \frac{\pi}{6} &= \sin^{-1}(0.4) \\ \frac{\pi}{2}x &= \sin^{-1}(0.4) + \frac{\pi}{6} \\ x &= \frac{2}{\pi}\left(\sin^{-1}(0.4) + \frac{\pi}{6}\right) = \frac{2}{\pi}\sin^{-1}(0.4) + \frac{1}{3}. \end{aligned}$$

$x_1 = \frac{2}{\pi}\sin^{-1}(0.4) + \frac{1}{3}$ $\quad x_2 = \frac{7}{3} - \frac{2}{\pi}\sin^{-1}(0.4)$ $\quad x_3 = x_1 + 4$