3. [10 points] Be sure to show your work on this problem. Parts **a.** and **b.** are not related.

a. [5 points] Consider the function $P = f(t) = 4 - e^{2t+5}$. Find $f^{-1}(P)$. Find a horizontal asymptote of $f(t)$.

$f^{-1}(P) =$ $\underline{\frac{\ln(4-P)-5}{2}}$.

A horizontal asymptote of $f(t)$ is $\underline{P = 4}$.

Solution: First, $f(t)$ is a shifted exponential function. Before the shift, the horizontal asymptote is $P = 0$ (all exponentials have this asymptote). But there is a reflection over the x-axis and a shift up 4 units, so the new asymptote is $P = 4$. To find the inverse, we need to solve for t, so we begin by subtracting 4 from both sides and then multiplying by -1 on both sides to get

$$4 - P = e^{2t+5}.$$

Taking ln on both sides gives us

$$\ln(4 - P) = 2t + 5.$$

Now we subtract 5 and divide by two

$$t = \frac{\ln(4 - P) - 5}{2}.$$

b. [5 points] Find all solutions x to the equation

$$4\cos(4x) = 1$$

on the interval $[-1, 2]$. Write your answer(s) in exact form. If there are no solutions, write "no solutions" in the blank and explain your answer.

$x =$ $\underline{\pm\frac{\arccos(1/4)}{4}, \frac{\pi}{2} \pm \frac{\arccos(1/4)}{4}}$.

Solution: The given equation is equivalent to

$$\cos(4x) = 1/4.$$

Looking on a calculator, we can see there are 4 intersection points between $y = \cos(4x)$ and $y = 1/4$. One of these is $x = \frac{\arccos(1/4)}{4}$. Let's abbreviate this solution θ Based on the symmetry of the cosine function, $-\theta$ is also a solution. The other two solutions correspond to the first two solutions each shifted one period ($\pi/2$) to the right, so all the solutions are:

$$x = \pm\frac{\arccos(1/4)}{4}, \frac{\pi}{2} \pm \frac{\arccos(1/4)}{4}.$$