3. [10 points] Be sure to show your work on this problem. Parts a. and b. are not related.

a. [5 points] Consider the function \( P = f(t) = 4 - e^{2t+5} \). Find \( f^{-1}(P) \). Find a horizontal asymptote of \( f(t) \).

\[
f^{-1}(P) = \frac{\ln(4-P) - 5}{2}.
\]

A horizontal asymptote of \( f(t) \) is \( P = 4 \).

Solution: First, \( f(t) \) is a shifted exponential function. Before the shift, the horizontal asymptote is \( P = 0 \) (all exponentials have this asymptote). But there is a reflection over the \( x \)-axis and a shift up 4 units, so the new asymptote is \( P = 4 \). To find the inverse, we need to solve for \( t \), so we begin by subtracting 4 from both sides and then multiplying by -1 on both sides to get

\[
4 - P = e^{2t+5}.
\]

Taking \( \ln \) on both sides gives us

\[
\ln(4 - P) = 2t + 5.
\]

Now we subtract 5 and divide by two

\[
t = \frac{\ln(4 - P) - 5}{2}.
\]

b. [5 points] Find all solutions \( x \) to the equation

\[
4 \cos(4x) = 1
\]
on the interval \([-1, 2]\). Write your answer(s) in exact form. If there are no solutions, write “no solutions” in the blank and explain your answer.

\[
x = \pm \frac{\arccos(1/4)}{4}, \frac{\pi}{2} \pm \frac{\arccos(1/4)}{4}.
\]

Solution: The given equation is equivalent to

\[
\cos(4x) = 1/4.
\]

Looking on a calculator, we can see there are 4 intersection points between \( y = \cos(4x) \) and \( y = 1/4 \). One of these is \( x = \frac{\arccos(1/4)}{4} \). Let’s abbreviate this solution \( \theta \) Based on the symmetry of the cosine function, \(-\theta \) is also a solution. The other two solutions correspond to the first two solutions each shifted one period \((\pi/2)\) to the right, so all the solutions are:

\[
x = \pm \frac{\arccos(1/4)}{4}, \frac{\pi}{2} \pm \frac{\arccos(1/4)}{4}.
\]