12. [10 points] Consider the rational function below where \( n \) is a positive whole number.

\[
Q(x) = \frac{(3x - 1)(x + 1)^2(x - 2)}{(x + 1)^n(x - 3)}.
\]

For each blank below, choose the best possible answer from the bottom of the page. There is only one best answer for each blank.

- **a.** [2 points] \( Q(x) \) has a hole at \( x = -1 \) ____________________________.

- **b.** [2 points] \( Q(x) \) has a vertical asymptote at \( x = -1 \) ____________________________.

- **c.** [2 points] \( Q(x) \) has no horizontal asymptotes ____________________________.

- **d.** [2 points] \( Q(x) \) has a horizontal asymptote at \( y = 0 \) ____________________________.

- **e.** [2 points] \( Q(x) \) has a vertical asymptote at \( x = \frac{1}{3} \) ____________________________.

**Possible answers:**

for any possible value of \( n \) for no possible values of \( n \)

for \( n \geq 2 \) for \( n \geq 3 \) for \( n \geq 4 \) for \( n = 1, 2 \) for \( n = 1, 2, 3 \)

for \( n = 1, 2, 3, 4 \) for \( n = 2, 3 \) for \( n = 2, 3, 4 \) for \( n = 3, 4 \)

for \( n = 1 \) only for \( n = 2 \) only for \( n = 3 \) only for \( n = 4 \) only

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