10. [11 points] After traveling back to present day, Kiki has given up on building time travel machines, but she is still building size-change machines and testing them out on her math notebooks each weighing 1kg. She has three machines with settings ranging from 1 to 100 (including non-whole number settings). On a setting of 8, each of the three machines changes the mass of a notebook to 5kg.

a. [3 points] On a setting of 38, the first machine changes the mass of the notebook to 3.5kg. Find a formula for $L(n)$, the mass of a notebook after being transformed by the first machine on a setting of n, if $L(n)$ is a **linear** function.

$L(n) =$ $\frac{-1}{20}(n-8)+5$

Solution: The slope is $(3.5-5)/(38-8) = \frac{-1}{20}$. We can then use point slope form and the point (8,5) to get the answer.

b. [4 points] On a setting of 10, the second machine changes the mass of the notebook to $\frac{20}{9}$kg. Find a formula for $E(n)$, the mass of a notebook after being transformed by the second machine on a setting of n, if $E(n)$ is an **exponential** function.

$E(n) =$ $\frac{5}{(2/3)^8}\left(\frac{2}{3}\right)^n$

Solution: If we use the form $E(n) = ab^n$, we can set up the equations

$$\frac{20}{9} = ab^{10}$$

and

$$5 = ab^8.$$

Dividing the first equation by the second, we get $\frac{4}{9} = b^2$, so $b = \frac{2}{3}$ (growth factor must be positive). Then using the second equation above, we get $a = \frac{5}{(2/3)^8}$.

c. [4 points] On a setting of 64, the third machine changes the mass of the notebook to $\frac{5}{4}$kg. Find a formula for $W(n)$, the mass of a notebook after being transformed by the third machine on a setting of n, if $W(n)$ is a **power** function.

$W(n) =$ $20n^{-2/3}$

Solution: If we use the form $W(n) = kn^p$, we can set up the equations

$$\frac{5}{4} = k64^p$$

and

$$5 = k8^p.$$

Dividing the first equation by the second, we get $\frac{1}{4} = 8^p$, so $p = -\frac{2}{3}$. Then using the first equation above, we get $k = 20$.