11. [11 points] The two parts of this problem are **unrelated**.

a. [6 points] Consider the quadratic function $y = f(x) = -3x^2 - x + 7$. By completing the square, find both coordinates of the vertex of this parabola in exact form. Show all steps of your calculation. Circle one of the options below the blank to indicate whether the vertex is a minimum or a maximum of the function.

The vertex is $\left(\frac{-1}{6}, \frac{85}{12}\right)$

and it's a:

$\boxed{\text{MAXIMUM}}$ MINIMUM

Solution: We complete the square:

$$\begin{aligned} y &= -3x^2 - x + 7 \\ &= -3\left(x^2 + \frac{1}{3}x\right) + 7 \\ &= -3\left(x^2 + \frac{1}{3}x + \frac{1}{36}\right) + 7 + \frac{1}{12} \\ &= -3\left(x + \frac{1}{6}\right)^2 + \frac{85}{12}. \end{aligned}$$

So the maximum is $\left(\frac{-1}{6}, \frac{85}{12}\right)$

b. [5 points] Consider the function $j(t) = 4 - 2e^{-t}$. Showing your work, find $j^{-1}(P)$ if it exists or explain why it does not exist.

$j^{-1}(P) =$ $-\ln\left(\frac{P-4}{-2}\right)$

Solution: This is a transformation of an exponential function, so it will pass the horizontal line test! To find the inverse we do the computation:

$$P = 4 - 2e^{-t}.$$

$$\frac{P-4}{-2} = e^{-t}.$$

$$-\ln\left(\frac{P-4}{-2}\right) = t.$$