2. [7 points]

a. [4 points] A population of fleas takes residence at the nearby *I-Love-Functions Dog Hotel* (oh no!) and the population grows exponentially for the first couple of days. At $t = 2$ hours after the infestation started, the population is 1000 fleas. By $t = 6$ hours after it started, the population is 2000 fleas. Write a formula for $P(t)$, the number of fleas t hours after the infestation started.
Show all work. Your final formula should include parameters in their EXACT form.

Solution: We know points on our function: $P(2) = 1000$ and $P(6) = 2000$. We also know that P is, for a while at least, an exponential function, so of the form: $P(t) = ab^t$, where a and b as as-of-yet unknown parameters. We can use the two point we know to set up two equations with two unknown parameters a, b:

$$2000 = a \cdot b^6$$

$$1000 = a \cdot b^2$$

One way to work with these equations and solve for one of the paramaters is to divide one equation by the other. Doing this we get:

$$2 = b^4$$

So $b = 2^{\frac{1}{4}}$. We can plug this back into either equation to solve for the value of a:

$$1000 = a \cdot (2^{\frac{1}{4}})^2$$

$$1000 = a \cdot 2^{\frac{1}{2}} = a\sqrt{2}$$

$$a = \frac{1000}{\sqrt{2}}$$

Putting these values back in for the parameters of $P(t)$ we get the final formula below.

$P(t) =$ $\frac{1000}{\sqrt{2}}(2^{\frac{1}{4}})^t$

b. [3 points] *Last* year a population of fleas also took up residence at the hotel and their population, as a function of hours since their arrival, was given by:

$$Q(t) = 500(1.22^t)$$

By what percent did *this* population increase each hour?

22 %

How long did it take for their initial population to triple?
Show all work. Give your final answer in decimal form, NOT exact form.

Solution: We are trying to find the value of t such that:$1500 = 500(1.22^t)$
We can solve this as follows:

$$\begin{aligned} 1500 &= 500(1.22^t) \\ 3 &= 1.22^t \\ \ln(3) &= \ln(1.22^t) \\ \ln(3) &= t\ln(1.22) \\ \ln(3)/\ln(1.22) &= t \\ 5.5248 &\approx t \end{aligned}$$

5.5248 hours