2. [7 points]

a. [4 points] A population of fleas takes residence at the nearby *I-Love-Functions Dog Hotel* (oh no!) and the population grows exponentially for the first couple of days. At $t = 2$ hours after the infestation started, the population is 1000 fleas. By $t = 6$ hours after it started, the population is 2000 fleas. Write a formula for $P(t)$, the number of fleas $t$ hours after the infestation started.

*Show all work. Your final formula should include parameters in their EXACT form.*

**Solution:** We know points on our function: $P(2) = 1000$ and $P(6) = 2000$. We also know that $P$ is, for a while at least, an exponential function, so of the form: $P(t) = ab^t$, where $a$ and $b$ as-as-of-yet unknown parameters. We can use the two point we know to set up two equations with two unknown parameters $a$, $b$:

\[
2000 = a \cdot b^6 \\
1000 = a \cdot b^2
\]

One way to work with these equations and solve for one of the parameters is to divide one equation by the other. Doing this we get:

\[
2 = b^4
\]

So $b = 2^{\frac{1}{4}}$. We can plug this back into either equation to solve for the value of $a$:

\[
1000 = a \cdot (2^{\frac{1}{4}})^2 \\
1000 = a \cdot 2^{\frac{1}{2}} = a\sqrt{2} \\
a = \frac{1000}{\sqrt{2}}
\]

Putting these values back in for the parameters of $P(t)$ we get the final formula below.

\[
P(t) = \frac{1000}{\sqrt{2}} \left(2^{\frac{1}{4}}\right)^t
\]

b. [3 points] *Last* year a population of fleas also took up residence at the hotel and their population, as a function of hours since their arrival, was given by:

\[
Q(t) = 500(1.22^t)
\]

By what percent did *this* population increase each hour?

\[
\frac{22}{\%}
\]

How long did it take for their initial population to triple?

*Show all work. Give your final answer in decimal form, NOT exact form.*
Solution: We are trying to find the value of $t$ such that: $1500 = 500(1.22^t)$

We can solve this as follows:

\[
\begin{align*}
1500 &= 500(1.22^t) \\
3 &= 1.22^t \\
\ln(3) &= \ln(1.22^t) \\
\ln(3) &= t \ln(1.22) \\
\frac{\ln(3)}{\ln(1.22)} &= t \\
5.5248 &\approx t
\end{align*}
\]

\[
\boxed{5.5248} \text{ hours}
\]