3. [8 points] Traditionally, it has been assumed that a $D$ year-old dog is the same biological age as a $7D$ year-old human. So a 3 year-old dog (in actual years) has aged as much as a 21 year-old human.

However, scientists have found a new aging formula for Labrador retrievers that takes specific biological aging markers into account. The new formula claims that a $D$ year-old Labrador retriever (in actual years) has aged as much as a human who is

$$H = f(D) = 15 \ln(D) + 31 \text{ years old}$$

One strange thing about this formula they came up with is that it doesn’t go through the point $(0,0)$ as we’d expect it to. In fact, we can’t plug in 0 to this formula at all!

a. [2 points] Explain in one sentence why we can’t plug $D = 0$ into this formula.

Explanation:

Solution: The domain of $\ln(D)$ does not include 0, so we can’t plug $D = 0$ into this formula. (This was enough explanation for full credit on the exam.)

(For further thought:) The reason the domain doesn’t include 0 is because $\ln$ is the inverse of the exponential function, and 0 isn’t in the range of $e^H$. That is, there is no value $H$ such that $e^H = 0$.

b. [3 points] According to this formula, at what age (in real years) will a dog be biologically equivalent to a newborn baby ($H = 0$)?

Show all work. Give your final answer in decimal form, NOT exact form.

Solution: You would expect that a 0-year old (newborn) dog would be equivalent to a 0-year old human (newborn). However, as we saw above, the formula doesn’t actually go through the point $(0,0)$ as we’d expect. In this question, we need to find the value of $D$ such that:

$$0 = 15 \ln(D) + 31$$

We can solve this algebraically:

$$\begin{align*}
0 &= 15 \ln(D) + 31 \\
-31 &= 15 \ln(D) \\
-\frac{31}{15} &= \ln(D) \\
\exp^{-\frac{31}{15}} &= D
\end{align*}$$

Putting this in decimal form using a calculator, we get that $D \approx 0.1266$. One way to interpret this would be that when a dog is 0.1266 years old, it is biologically equivalent to a newborn human. (It makes sense that this number is small!)

$$D = \boxed{0.1266} \text{ years}$$

This problem continues on the next page.
c. [3 points] Now considering the same function without its context: which of the graphs below could be the graph of

\[ f(D) = 15\ln(D) + 31? \]

Circle the correct graph or None.

None of these graphs could represent the function \( f(D) \).

Solution: The graph of \( f(D) \) is related to the graph of \( \ln(D) \). It is the original graph stretched vertically, then shifted up. Neither of these changes affect the position of the original vertical asymptote of \( D = 0 \), so the only possible graph is the one in the upper left, which is the same basic shape as the graph \( \ln(D) \) function, with the vertical asymptote still at \( D = 0 \).

The upper right graph is a shift of \( e^D \) (not a \( \ln \) function at all), and the two lower graphs are \( \ln(D) \) functions that include a shift right or left.