3. [8 points] Traditionally, it has been assumed that a D year-old dog is the same biological age as a $7D$ year-old human. So a 3 year-old dog (in actual years) has aged as much as a 21 year-old human.

However, scientists have found a new aging formula for Labrador retrievers that takes specific biological aging markers into account. The new formula claims that a D year-old Labrador retriever (in actual years) has aged as much as a human who is

$$H = f(D) = 15\ln(D) + 31 \text{ years old}$$

One strange thing about this formula they came up with is that it doesn't go through the point $(0,0)$ as we'd expect it to. In fact, we can't plug in 0 to this formula at all!

a. [2 points] Explain in one sentence why we can't plug $D = 0$ into this formula.

Explanation:

Solution: The domain of $\ln(D)$ does not include 0, so we can't plug $D = 0$ into this formula. (This was enough explanation for full credit on the exam.)

(For further thought:) The reason the domain doesn't include 0 is because ln is the inverse of the exponential function, and 0 isn't in the *range* of e^H. That is, there is no value H such that $e^H = 0$.

b. [3 points] According to this formula, at what age (in real years) will a dog be biologically equivalent to a newborn baby ($H = 0$)?
Show all work. Give your final answer in decimal form, NOT exact form.

Solution: You would expect that a 0-year old (newborn) dog would be equivalent to a 0-year old human (newborn). However, as we saw above, the formula doesn't actually go through the point $(0,0)$ as we'd expect. In this question, we need to find the value of D such that: $0 = 15\ln(D) + 31$. We can solve this algebraically:

$$\begin{aligned} 0 &= 15\ln(D) + 31 \\ -31 &= 15\ln(D) \\ \frac{-31}{15} &= \ln(D) \\ e^{\frac{-31}{15}} &= D \end{aligned}$$

Putting this in decimal form using a calculator, we get that $D \approx 0.1266$. One way to interpret this would be that when a dog is 0.1266 years old, it is biologically equivalent to a newborn human. (It makes sense that this number is small!)

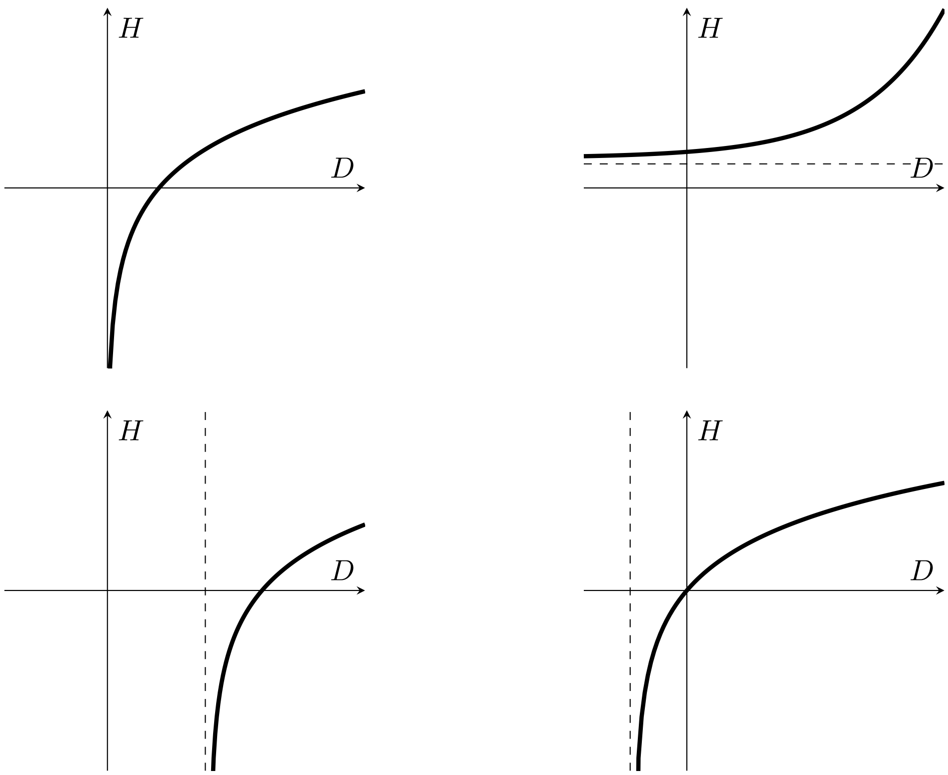
$D =$ **0.1266** years

This problem continues on the next page.

c. [3 points] Now considering the same function without its context: which of the graphs below could be the graph of

$$f(D) = 15\ln(D) + 31?$$

Circle the correct graph or NONE.



NONE OF THESE GRAPHS COULD REPRESENT THE FUNCTION $f(D)$.

Solution: The graph of $f(D)$ is related to the graph of $\ln(D)$. It is the original graph stretched vertically, then shifted up. Neither of these changes affect the position of the original vertical asymptote of $D = 0$, so the only possible graph is the one in the upper left, which is the same basic shape as the graph $\ln(D)$ function, with the vertical asymptote still at $D = 0$.

The upper right graph is a shift of e^D (not a ln function at all), and the two lower graphs are $\ln(D)$ functions that include a shift right or left.