8. [5 points] The *I-Love-Functions Dog Hotel* has a one-of-a-kind Doggie Ferris Wheel for its residents to use on special occasions. The hotel residents board the Doggie Ferris Wheel at its lowest point, from a platform that is 5 feet high. The Doggie Ferris Wheel is 34 feet in diameter.

a. [3 points] If each full rotation rotation takes 1 minute, how high off of the ground is a dog when she is exactly 20 seconds into the ride?
Show all work (including any pictures). Give your final answer in decimal form, NOT exact form.

Solution: There were multiple ways to approach this problem! One way was to first model the height of a dog using a sinusoidal function. Since this function starts at its minimum value (when the dog boards), our simplest starting function will be $\cos(x)$. Then factoring in the amplitude, period, and shift, we end up with the following height formula as a function of time t in seconds:

$$h(t) = -17\cos\left(\frac{2\pi}{60}t\right) + 22$$

To compute the dogs height at 20 seconds, we can plug $t = 20$ into the above formula and get $h(20) = 30.5$.

We need to consider whether or calculator should be in radians or degrees when we do this. The coefficient $2\pi/60$ was implicitly assuming we were starting with a cos function with period 2π, so our calculator should be in radians to use this formula. If we didn't realize that and it was in degrees, then we'd get 5.011 feet. However, since they're boarding at 5 feet and going all the way up to 39 feet, it doesn't make sense that 20 seconds into a 60-second rotation would have the dog just slightly above the boarding height—so we could catch our own mistake this way.

There is a totally different way of answer this problem. Consider a Ferris Wheel with center $(0, 22)$ and let's figure out where on the Ferris Wheel we'd be at 20 seconds. Since 20 seconds is a third of a 60-second rotation, it would put the dog at $360°/3 = 120°$ from the boarding point, so at 30° above the horizontal point ("3 o'clock position"). The height at this angle above the "3 o'clock position" will be $\sin(30°) \cdot 17 = \frac{1}{2} \cdot 17 = 8.5$ feet. If we add this to a starting height at the 3 o'clock position of 22 feet, we get the same height as the method above: 30.5 feet.

Height: **30.5** feet

b. [2 points] What length of the Doggie Ferris Wheel's arc is traversed by a passenger dog in 47 seconds of riding?
Show all work (including any pictures). Give your final answer in decimal form, NOT exact form.

Solution: The total circumference of the Ferris Wheel is $2\pi \cdot 17 = 34\pi \approx 106.81$ feet. The dog is riding for $\frac{47}{60}$ of a full rotation, so a total of $\frac{47}{60} \cdot 34\pi = 83.671$ feet.

Length of arc: **83.671** feet