2. [9 points] The parts of this problem are unrelated.

a. [6 points] Consider the quadratic function $f(x) = (x-1)^2 - 1$.

(i) Find the zero(s) and vertex of $f(x)$. *Show any relevant work.*

Solution: $f(x)$ is already given to us in vertex form, so we know that the vertex of $f(x)$ is $(1,-1)$. To find the zeros, we have a few options. One path is that we can set up the following equation and solve:

$$\begin{aligned} 0 &= (x-1)^2 - 1 \\ 1 &= (x-1)^2 \\ \pm 1 &= x - 1 \\ 1 \pm 1 = 0, 2 &= x \end{aligned}$$

vertex: $(1,-1)$ zero(s): $x =$ 0, 2

(ii) Find the vertex of $f(2x) + 1$.

Solution: Because we we are given a transformation of f, we can identify what transformation took place, then apply those to the original vertex. The transformations that took place are a horizontal compression by $\frac{1}{2}$ and a vertical shift by 1. This means that our new vertex is at $(\frac{1}{2}, 0)$.

vertex: $(0.5, 0)$

(iii) Find the zero(s) of $3f(x-1)$.

Solution: Again, we need to consider what transformation have been applied, graphically, to the original function f. In this case, there has been a vertical stretch by a factor of 3, and a horizontal shift right by 1. The vertical stretch will not change the value of the zeros, since they will remain zero under a vertical stretch. But the shift right will do just that, shift them right. So our new zeros will be $x = 1, 3$

zero(s): $x =$ 1, 3

b. [3 points] A different quadratic function, $g(x)$, has its vertex at $(2,3)$ and a zero at $x = -1$.

(i) What is the x-coordinate of the other zero of $g(x)$? (CIRCLE ONE)

0 1 2 3 4 $\boxed{5}$ NONE OF THESE NOT ENOUGH INFORMATION

Solution: Any parabola has a vertical axis of symmetry. In this case, this axis of symmetry is given by $x = 2$, which we know from the given vertex. Any zeros are equidistance from the axis of symmetry. One zero is at -1, which is a distance of 3 away from $x = 2$. Thus the second zero is a distance 3 away from $x = 2$ in the other direction. In otehr words, it's $x = 5$.

(ii) What is the sign of the leading coefficient of $g(x)$? (CIRCLE ONE AND EXPLAIN)

$\boxed{\text{negative}}$ positive zero NOT ENOUGH INFORMATION

Explanation of (ii):

Solution: Because the vertex is *above* the zeros, the parabola must be "downward facing" or concave down. This means that its leading coefficient must be negative.