4. [11 points] As stated in the problem above, the price of each t-shirt $P(s)$ (in dollars) at *Amaizing T-Shirts* is a function of s, the total number of t-shirts a customer orders. In particular, now assume that if the customer orders exactly 1 t-shirt, it costs \$14.50. If the customer orders 30 t-shirts, each shirt costs \$13.30.

a. [3 points] If we assume that $P(s)$ is a **linear function**, find a formula for $P(s)$ *Show all work. Numbers in your final function can be rounded to two decimal places or expressed in exact form.*

Solution: We are essentially looking for a linear equation that goes through the points $(1, 14.5)$ and $(30, 13.30)$. First, we can compute the slope:

$$m = \frac{13.30 - 14.50}{30 - 1} = \frac{-1.20}{29} \approx -0.04138$$

Then, one of the simplest ways to complete our linear equation is to use point-slope form, based off of either given point. One such solution is shown below.

$P(s) = \frac{-1.2}{29}(s-1) + 14.5 \approx -0.041s + 14.54$

b. [2 points] What is the meaning of the slope of $P(s)$ in your linear function above?

Meaning of Slope:

Solution: The slope found above represents the change in price-per-shirt for each additional shirt added to the total. That is, for each additional shirt added to the total purchase, the price-per-shirt will decrease by a little less than 4 cents.

c. [3 points] If we assume that $P(s)$ is an **exponential function**, find a formula for $P(s)$ *Show all work. Numbers in your final function can be rounded to two decimal places or expressed in exact form.*

Solution: We are essentially looking for an exponential equation that goes through the points $(1, 14.5)$ and $(30, 13.30)$. That is, we're looking for a function of the form ab^s. We can use our two known points to come up with two different equations, and then solve for the unknown parameters a, b.

$$13.30 = a \cdot b^{30} \qquad 14.50 = a \cdot b^1 = ab$$

If we divide the left equation by the right we get:

$$\frac{13.3}{14.5} = \frac{ab^{30}}{ab} = b^{29}$$

Thus $b = \left(\frac{13.3}{14.5}\right)^{\frac{1}{29}} \approx 0.997$
We can use that value for b to then solve for a:

$$14.50 = a \cdot \left(\frac{13.3}{14.5}\right)^{\frac{1}{29}}$$

so $a = 14.5 \cdot \left(\frac{13.3}{14.5}\right)^{-\frac{1}{29}} \approx 14.543$

$P(s) = 14.5\left(\frac{13.3}{14.5}\right)^{-\frac{1}{29}} \cdot \left(\frac{13.3}{14.5}\right)^{\frac{s}{29}} \approx 14.543 \cdot 0.997^s$

d. [3 points] If we assume that $P(s)$ is a **power function**, find a formula for $P(s)$ *Show all work. Numbers in your final function can be rounded to two decimal places or expressed in exact form.*

Solution: We are essentially looking for a power function equation that goes through the points $(1, 14.5)$ and $(30, 13.30)$. That is, we need to find the parameters k and p so that $y = ks^p$ will go through the given points. We can use our two known points to come up with two different equations, and then solve them for the unknown parameters.

$$13.30 = k \cdot 30^p \qquad 14.50 = k \cdot 1^p = k$$

Therefore we know right away that $k = 14.5$. We can use that then to solve for p in the first of the two equations:

$$13.30 = 14.5 \cdot 30^p$$
$$\frac{13.3}{14.5} = 30^p$$
$$\log\left(\frac{13.3}{14.5}\right) = p\log(30)$$
$$\frac{1}{\log(30)} \cdot \log\left(\frac{13.3}{14.5}\right) \approx -0.025 = p$$

A negative value for p makes sense, because our function is decreasing. Our final formula can be seen below:

$P(s) =$ $14.5s^{\frac{1}{\log(30)} \cdot \log\left(\frac{13.3}{14.5}\right)} \approx 14.5s^{-0.025}$