7. [10 points] Madeline is the head engineer of a new racetrack called the "Michigan Raceway". Unusually, this track will be a perfect circle. The radius a car's path will be 800m.

a. [2 points] While driving the test car, Madeline drives at a perfectly constant speed and makes one complete revolution in 1.5 minutes. How far, in meters, does Madeline drive in 40 seconds? *Show all work. Give your answer in exact form, or rounded to two decimal places.*

Solution: In 40 seconds she would have driven 40/90 = 4/9 of a rotation. Since a full circumference is 1600π meters, she would have driven $\frac{4}{9} \cdot 1600\pi \approx 2234.021$ meters in 40 seconds.

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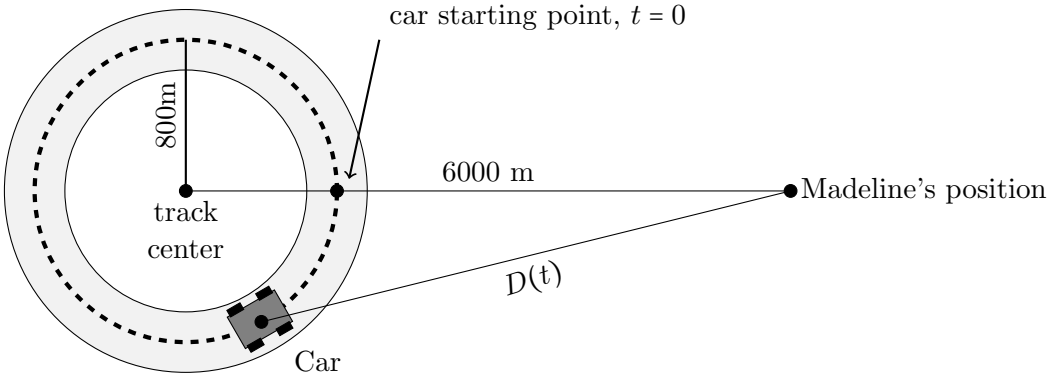
b. [4 points] While another engineer takes over the test car, Madeline stands directly east, 6000m away from **center** of the track. The car now drives at a constant speed and takes exactly 2 minutes for each lap. Let $D(t)$ be the test car's distance from Madeline, in meters, where t is measured in minutes since the car started, directly east of center.



(i) What is the minimum value of $D(t)$? 5200 meters

(ii) What is the maximum value of $D(t)$? 6800 meters

(iii) What is the exact value of $D(0.5)$? *Show all work.*

Solution: At 0.5 minutes, the car is exactly one quarter into a rotation, so either at the far north point of the track shown, or the far south point (depending on which way the car is going). In either case, the car's distance from Madeline can be found as the length of the hypotenuse of a right triangle. One of its side lengths is 800m, the other is 6000m. So the hypotenuse length (and hence $D(0.5)$) is

$$\sqrt{800^2 + 6000^2} \approx 6053.10 \text{ meters}$$

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c. [4 points] If we want to *approximate* a formula for $D(t)$ using a sinusoidal function of the form $D(t) = A\cos(B(t-h))+k$, what formula should we use? That is, use the information above to find the appropriate values for A, B, h, and k and write out the final formula.

$D(t) =$ $-800\cos(\pi t) + 6000$