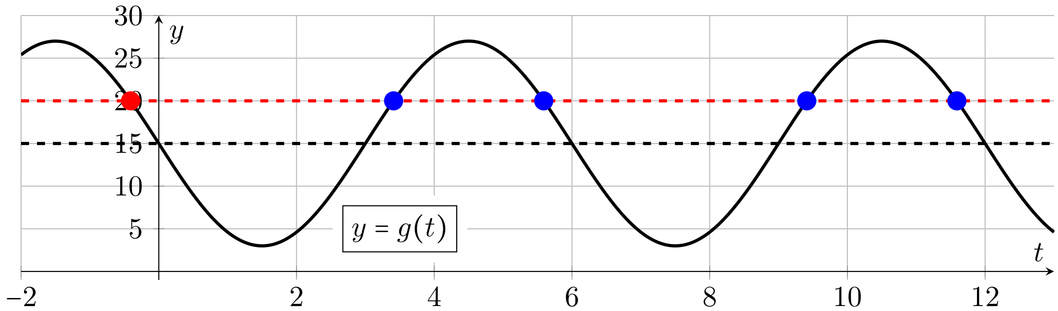
9. [8 points] The following problem parts are not related.

a. [3 points] The function $g(t)$, shown in the graph below, is a sinusoidal function with

- period 6
- midline $y = 15$
- and y-intercept $(0, 15)$

Using the fact that $g(-0.41) = 20$, find all other solutions to $g(t) = 20$ on the domain $[0, 12]$ and illustrate on the graph where they fall using dots.



Solution: We added the given solution with a red dot in the graph above. We know that two of the needed solutions will be one period, and two periods, respectively, further in the positive direction. Those will be $-0.41 + 6 = 5.59$ and $-0.41 + 12 = 11.59$. These are are the second and fourth dots from the left in the diagram above. To find the t-coordinates of the remaining solutions (the 1st and 3rd blue dots from the left), we need to use graph symmetry. There are several ways to approach that. One is to notice that the given solution, in red, is as far *before* $t = 0$ as the next solution is *after* $t = 3$ (the next time the graph crosses the midline). This makes that solution 3.41. And then a last solution is one period later: $3.41 + 6 = 9.41$.

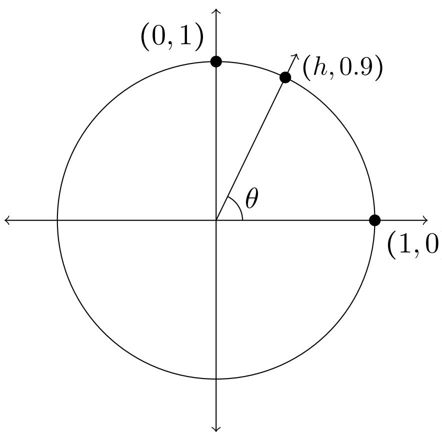
$t =$ **3.41, 5.59, 9.41, 11.59**

b. [5 points] In the unit circle shown below, the ray at angle θ to the positive x-axis intersects the unit circle at the coordinates $(h, 0.9)$.

(i) What is the value of h? *Show all relevant work. Give your final answer in exact form, or accurate to two decimal places.*

Solution: $h^2 + (0.9)^2 = 1 \implies h = \sqrt{1-(0.9)^2} \approx 0.44$

$h =$ $\sqrt{1-(0.9)^2} \approx 0.44$



(ii) What is the value of θ in degrees? *Show all relevant work. Give your final answer in **numerical** form, accurate to two decimal places.*

Solution: $\sin(\theta) = 0.9 \implies \theta = \arcsin(0.9) = 64.16°$, or $180° - 64.16°$. Since the angle is in the first quadrant, the correct answer is $64.16°$.

$\theta =$ 64.16 $\circ$

(iii) Find the value for an angle ϕ (in degrees), between 90° and 180°, such that $\sin(\phi) = 0.9$. *Show all relevant work. Give your final answer in terms of θ or as a number rounded to two decimal places.*

Solution: $\sin(\theta) = 0.9 \implies \theta = \arcsin(0.9) = 64.16°$, or $180° - 64.16°$. Since the angle is in the second quadrant, the correct answer is $180° - 64.16°$.

$\phi =$ 115.84 $\circ$

(iv) Find **all** possible values of an angle ω (in degrees) between 0 and 360° such that $\cos(\omega) = 0.9$. *Show all relevant work. Give your final answer in terms of θ or as a number rounded to two decimal places.*

Solution: **Solution 1.** $\cos(\omega) = 0.9 \implies \omega = \arccos(0.9) = 25.84° + 360°k$, or $360°k - 25.84°$. The only angles among these that are in the interval 0° to 360° are 25.84° and $360° - 25.84°$.

Solution 2. At the angle θ, $\sin(\theta) = 0.9$ so $\cos(90° - \theta) = 0.9$. If $\cos(\omega) = 0.9$, then $\cos(-\omega) = 0.9$ also, since cos is even. So $\theta - 90°$ is also a solution, but it's not in the interval 0° to 360°, so we add 360° to get $\theta + 270°$.

$\omega =$ $25.84°$**,** $360° - 25.84°$ **or** $90 - \theta$**,** $\theta + 270$ $\circ$