2. [11 points] Invasive beetles were accidentally introduced to a nature preserve, and their population then grew exponentially for 11 weeks. In particular, the number of beetles in the preserve t weeks after their introduction was modeled by the function

$$b(t) = 4(1.5)^t \text{ for } 0 \le t < 11.$$

Show your work and give answers in exact form or rounded to at least two decimal places unless otherwise noted.

a. [3 points] By what percent did the beetle population grow each **day**?

Solution: Since $t = 1/7$ corresponds to one day, we can find $b(1/7) = 4(1.5)^{1/7}$. Dividing this number by 4 to find the percent increase from the initial value, we get $(1.5)^{1/7} \approx 1.0596$.

Answer: $100\left((1.5)^{1/7} - 1\right) \approx 5.96$ %

b. [3 points] At what time t was the number of beetles equal to 100?

Solution: We solve $4(1.5)^t = 100$ to find that $(1.5)^t = 25$, so $t = \dfrac{\ln(25)}{\ln(1.5)}$.

Answer: $t =$ ≈ 7.94

c. [5 points] The beetle was detected and, after 11 weeks, eradication efforts began. From that time, the population decreased at a rate of 50 beetles per week until the population was completely removed.

i. How many beetles were there after 11 weeks?
You may round your answer to the nearest whole beetle.

Answer: $4(1.5)^{11} \approx 346$ beetles

ii. From the time the beetle population was introduced to the preserve, how many weeks passed before it was completely removed?

Solution: When eradication efforts began, there were 346 beetles, so it took $346/50 \approx 6.92$ weeks beyond the first 11 weeks.

Answer: 17.92 weeks

iii. Use your answers to complete the piecewise formula given below for the beetle population $b(t)$ from when it was first introduced to the preserve until the time it was completely removed.

Answer: $$b(t) = \begin{cases} 4(1.5)^t & 0 \le t < 11 \\ 346 - 50(t - 11) & \mathbf{11} \le t \le \mathbf{17.92} \end{cases}$$