4. [10 points] Rupert the hamster has another new hamster wheel. It has a diameter of 9 inches and lies 2 inches off the ground. Consider the point (0,0) to be on the ground directly below the center of the wheel, as shown. Rupert's owner adds a dot to the wheel so she can track how much Rupert runs.

Just before Rupert starts running, the dot makes an angle of $\theta = \frac{\pi}{4}$ with the positive horizontal axis.

Show your work and give all answers in exact form or rounded to at least two decimal places.

a. [3 points] Find the coordinates of the dot in its current position.

Answer: $(4.5\cos(\frac{\pi}{4}) \approx 3.18)$, $4.5\sin(\frac{\pi}{4}) + 6.5 \approx 9.68$

Rupert begins to run.

- The wheel turns counterclockwise, so that the dot moves from its location in the diagram up and left before continuing around the wheel.
- He runs at a constant pace so that each full turn of the wheel takes 4 seconds.
- The dot travels 3 full rotations, followed by a partial rotation so that it ends up at the **bottom** of the wheel when Rupert stops running.
- **b**. [2 points] How many rotations of the wheel did the dot make?

Answer: $3 + \frac{5}{8} = 3.625$

c. [1 point] For how many seconds did Rupert run?

Solution: We take the number of rotations times 4 seconds per rotation.

Answer: <u>14.5</u> seconds

d. [2 points] What distance did Rupert run?

Solution: We can take the number of rotations times the circumference of 9π , or the radius 4.5 times the angle in radians, $2\pi * 3 + \frac{5\pi}{4}$.

Answer: $9\pi \cdot \frac{29}{8} \approx 102.49$ inches While jumping off, Rupert causes the wheel to rotate another $\frac{\pi}{6}$ radians counterclockwise. e. [2 points] What angle does the dot now make with the positive horizontal axis?

Answer: $-\frac{\pi}{3}$ or $\frac{5\pi}{3}$ radians

