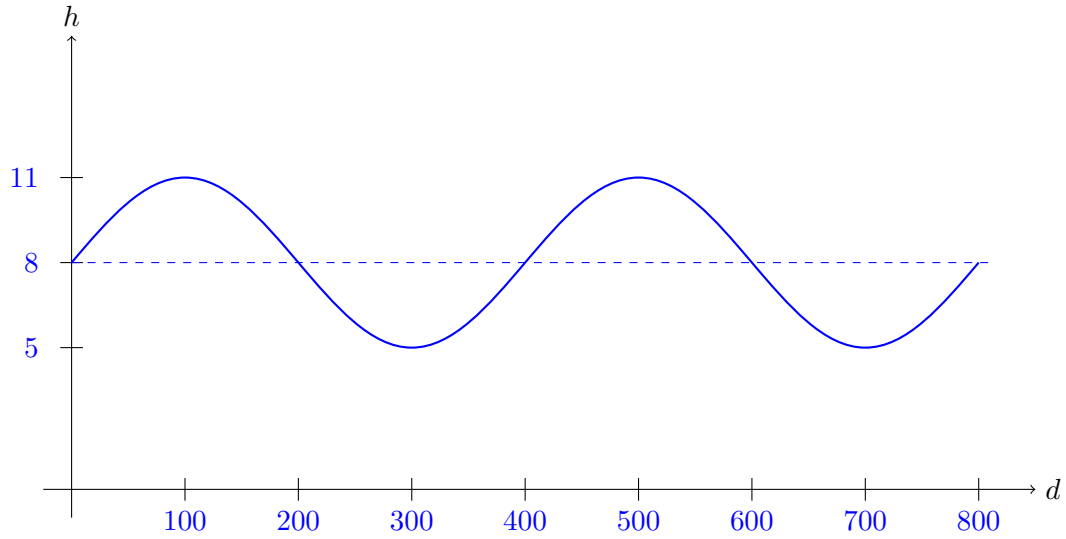
6. [14 points] On the planet of Sinosia, the number of hours of direct sunlight at a particular location varies sinusoidally throughout the year (which is not the same length as a year on Earth). In particular, the number of hours of daylight on the dth day is given by

$$S(d) = 8 + 3\sin\left(\frac{\pi}{200}d\right).$$

a. [4 points] On the axes below, sketch a graph of **two periods** of $h = S(d)$. Your second cycle should end at the d-value indicated by the tick mark furthest to the right. Clearly label at least two of the tick marks on the d-axis. On the h-axis, add and label at least two tick marks to indicate the maximum and minimum values of $S(d)$.



b. [5 points] Find the first 3 positive d values for which there is 10 hours of direct sunlight. *Show your work and give answers in exact form or rounded to the nearest day.*

Solution: We solve the equation $S(d) = 10$ for d algebraically to find the two fundamental solutions:

$$\begin{aligned} 8 + 3\sin((\pi/200)d) &= 10 \\ \sin((\pi/200)d) &= 2/3 \\ (\pi/200)d &= \arcsin(2/3) \\ \text{or } (\pi/200)d &= \pi - \arcsin(2/3). \end{aligned}$$

This leads to solutions of $d = (200/\pi)\arcsin(2/3) \approx 46$
and $d = (200/\pi)(\pi - \arcsin(2/3)) = 200 - (200/\pi)\arcsin(2/3) \approx 154$.

Then the third solution must be one period after the first, so $d = (200/\pi)\arcsin(2/3) + 400 \approx 446$.

Answer: $d =$ **46**, **154**, **446**

(Problem continues on the next page.)

On the nearby planet of Cosinia, the number of hours of direct sunlight also varies sinusoidally throughout its year.

- On the 30th day, the amount of sunlight reaches a peak of 16 hours.
- On the 80th day, the amount of sunlight is at its minimum of 4 hours.

Let $C(d)$ represent the number of hours of daylight on the dth day.

c. [1 point] How many days are there on Cosinia per year? In other words, what is the period of the function $C(d)$?

Answer: 100 days

d. [4 points] Find a formula for the function $C(d)$.

Answer: $C(d) = 6\cos\left(\frac{2\pi}{100}(d-30)\right)+10$