MATH 115 — FINAL EXAM

December 20, 2004

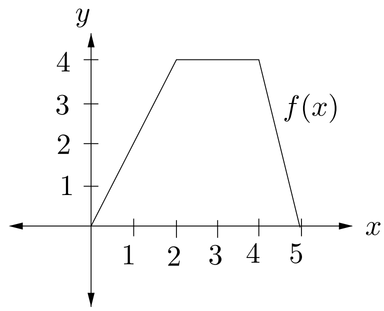
NAME: ____

INSTRUCTOR: ______________________ SECTION NO: __________

1. Do not open this exam until you are told to begin.
2. This exam has 10 pages including this cover. There are 10 questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator. You are also allowed 2 sides of a 3 by 5 note card.
7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to make clear how you arrived at your solution.
8. Please turn **off** all cell phones and all other sound devices. Remove all headphones.

PROBLEM	POINTS	SCORE
1	12	
2	6	
3	5	
4	8	
5	13	
6	9	
7	15	
8	9	
9	10	
10	13	
TOTAL	100	

1. (12 points) Let $g(x)$ be a continuous function such that $\int_2^3 g(x)dx = 5$. Let $f(x)$ be given by the following graph:



(a) Find $f'(1)$.

(b) Find $\int_1^2 g(x+1)dx$.

(c) Find the average value of f on the interval $[0, 4]$.

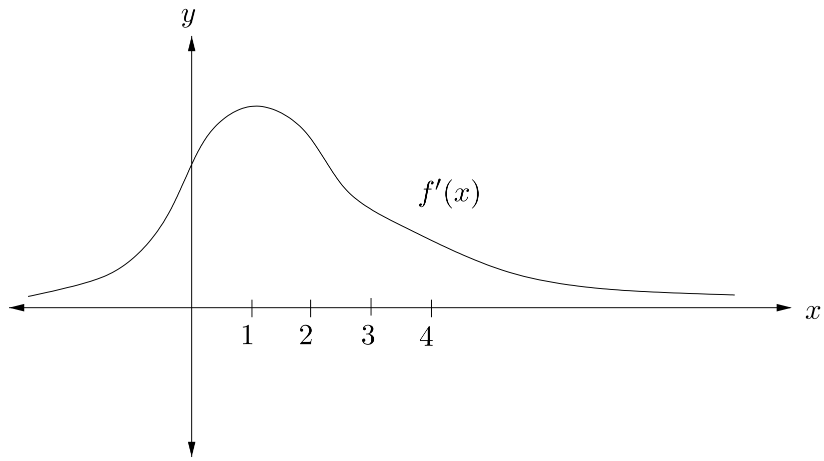
(d) Find $\int_2^3 (f(x) + 3g(x))dx$.

(e) If $G'(x) = g(x)$ and $G(2) = 7$, find $G(3)$.

(f) If $F'(x) = f(x)$, describe two graphical features of F on the interval $0 < x < 1$.

2. (6 points) Using the graph of $f'(x)$ provided, list the following in increasing order:

$$\int_1^3 f'(x)dx, \qquad f(3) - f(2), \qquad f(2) - f(1)$$

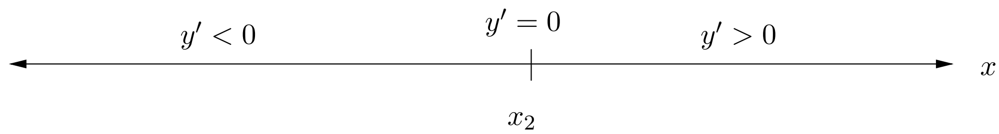


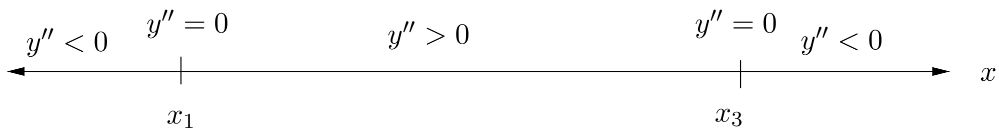
______________ less than ______________ less than ______________

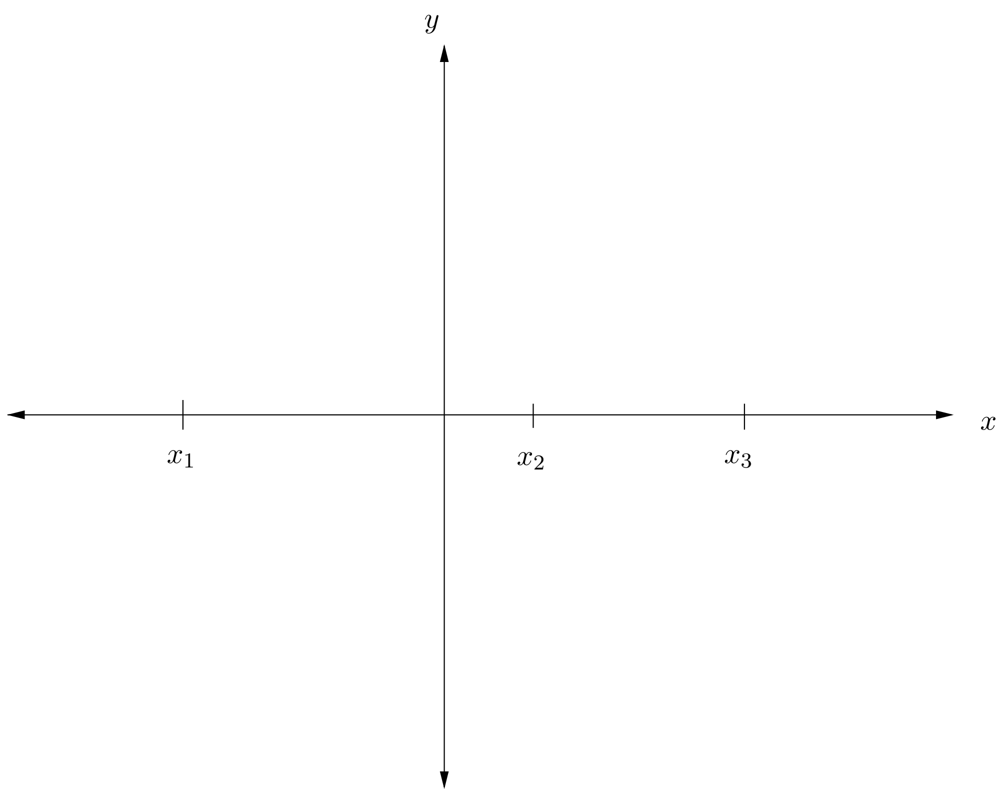
3. (5 points) **(a)** Briefly explain the difference between the indefinite integral $\int f(x)\,dx$ and the definite integral $\int_a^b f(x)\,dx$.

(b) What is the connection between a Riemann sum and one or more of the integrals in part (a)? Your answer should include a picture and a clear explanation.

4. (8 points) Sketch a possible graph of $y = f(x)$, using the given information about the derivatives $y' = f'(x)$ and $y'' = f''(x)$. Assume that f is defined and continuous for all real x. Label all local extrema and inflection points.







5. (4+6+3 points) Your uncle Harry absolutely LOVES eggnog around the holidays. The rate at which he drinks it at your family holiday party is given by the function $r(t)$ where t is measured in hours and $r(t)$ is in liters/hour. Suppose $t = 0$ corresponds to 6 pm when the party begins.

(a) Write a definite integral that represents the total amount of eggnog uncle Harry consumes between 8 pm and 2 am the next morning.

(b) If Uncle Harry's rate of eggnog drinking is given by $r(t) = e^{-t} + 1$, use a left hand sum with three (3) subdivisions to estimate the amount of nog Harry drinks in the first four hours of the party. Show all of your work.

(c) Should your estimate in part (b) be an underestimate or an overestimate? Explain.

6. (9 points) You feel that you need to escape the cold climate before you return to school, so you decide to drain your saving account and head for somewhere warm and tropical. However, the cost of airline tickets seems to vary from day to day, moment to moment. At the eggnog party you meet a person who works in airline ticket pricing. He actually gives you a web site where you can check out the rate at which the cost of tickets on your preferred airline is changing at any given moment on a given day. (At this site, the rate of change is defined for all times, t.)

(a) If the rate of change is negative, would you be inclined to buy or wait? Why?

(b) If the rate of change is equal to zero, what would that mean? Explain.

(c) If you could get the formula for the data generated on the web site, would that help you to decide what to do in the case of part (b)? If so, how? If not, why not?

7. (4+2+3+6 points) You got a flight out, finally, but in your haste to leave, you locked Frosty the Snowman, Jr. in the warm greenhouse. Suppose $r(t)$ is the rate in cm^3/min that the Frosty's volume is changing as he is trapped in the greenhouse. The time the doors of the greenhouse were closed corresponds to $t = 0$.

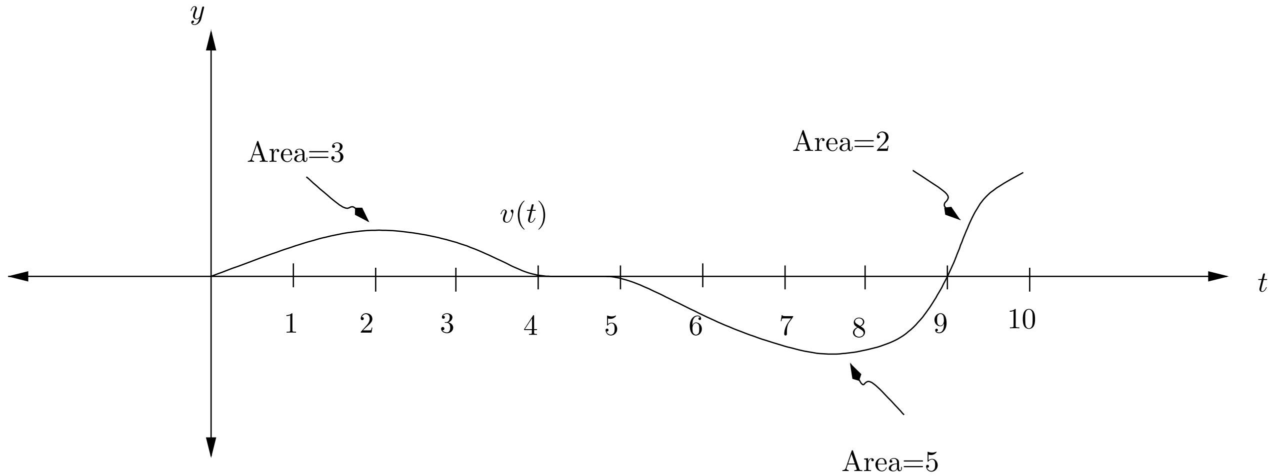
(a) Explain the meaning of the quantity $\int_2^5 r(t)dt$ in the context of this problem.

(b) What do you expect the sign of $r(t)$ to be for the meaningful domain of this problem? Why?

(c) If $r(t) = 3t^2 - 432$, what is the domain that makes sense for this problem? Why?

(d) Use the Fundamental Theorem of Calculus (and common sense) to determine the volume (in cm^3) of Frosty, Jr. when the door to the greenhouse was closed. Show all of your work and reasoning.

8. (3+3+3 points) Finally in your tropical paradise, you are strolling through the rain forest when you come upon a hummingbird. He is flitting up and down a vine of flowers. The graph below gives the bird's **vertical** velocity (ft/sec) as a function of time (sec). Positive velocity indicates he is going up, and negative velocity indicates down.

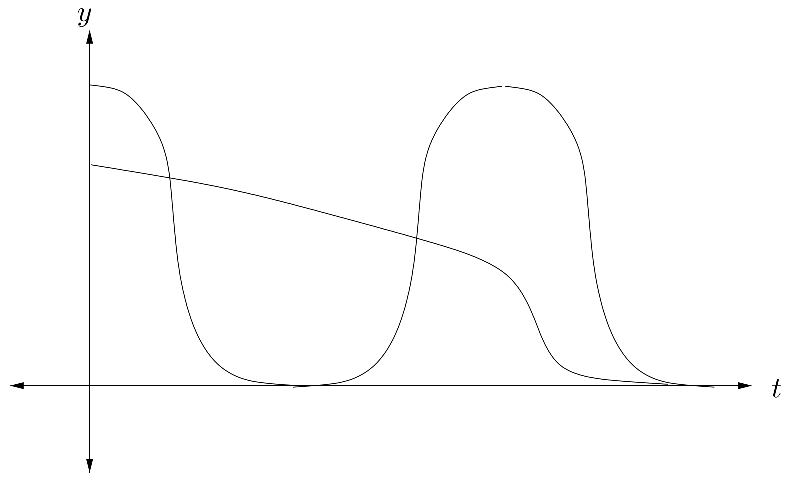


(a) At which time(s) is the hummingbird likely hovering at a flower? Explain how you arrived at your answer.

(b) At which time during the 10-second period is the hummingbird highest off the ground? Explain how you arrived at your answer.

(c) At which time(s) is the hummingbird's vertical acceleration the greatest? Explain how you arrived at your answer.

9. (4+3+3 points) After watching hummingbirds for a while, you head to the beach to tan. Shortly after you begin tanning you are overcome with boredom. You decide to pass your time by digging a glorious hole in the ground. Just as a satisfactory hole begins to emerge from your efforts, the tide comes in and waves begin to wash sand back into your hole. You continue digging, but your enthusiasm is ending. Finally, you realize you're defeated and give up. The rate of sand flow into the hole due to the incoming waves and the rate of flow of sand out of the hole due to your digging are graphed below. Assume time $t = 0$ corresponds to the first wave coming in.



(a) Label which curve corresponds to the rate of sand going out of the hole and which curve corresponds to the rate of sand being washed into the hole by the waves. Explain how your arrived at your answer.

(b) Mark and label the point on the graph when the least amount of sand was in the hole. Label the point as A. Explain how you arrived at your answer.

(c) Mark and label the point on the graph when the amount of sand in the hole was increasing most rapidly? Label the point as B. Explain how you arrived at your answer.

10. (13 points) Alas, you returned home without a cent to your name–but you had a great time! You are hitching a ride back to school with friends. You have agreed to take a small suitcase in order to save space, but you realize that you also need to take all the souvenirs that you are bringing back to your friends at school. You determine that you need 3000 in^3 of space for those gifts. Your friend agrees to strap a box to the roof of his car. You are worried about the straps crushing the box and about the weather, so you decide to have a box made. The box-making place agrees to make a rectangular box with reinforced sides and weatherproofing on the top and sides. The sides come in a standard height of 6 inches and cost \$0.25 per in^2. The top costs \$0.15 per in^2, and the materials for the bottom cost \$0.05 per in^2. There is also a \$10 charge for labor.

(a) You are going to have to go into debt to get this box, so find the dimensions of the box with the standard height sides (6 inches) that will minimize the cost.

(b) How much is the box going to cost?

May you all have a wonderful holiday, wherever you go!