MATH 115 — FIRST MIDTERM EXAM

October 11, 2005

NAME: ____

INSTRUCTOR: ______________________ SECTION NO: __________

1. Do not open this exam until you are told to begin.
2. This exam has 9 pages including this cover. There are 10 questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator. You are also allowed 2 sides of a 3 by 5 note card.
7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to make clear how you arrived at your solution.
8. Please turn **off** all cell phones and other sound devices, and remove all headphones.

PROBLEM	POINTS	SCORE
1	12	
2	12	
3	4	
4	12	
5	12	
6	10	
7	12	
8	12	
9	5	
10	9	
TOTAL	100	

1. (2 points each, no partial credit) Circle "True" or "False" for each of the following problems. Circle "True" only if the statement is *always* true. No explanation is necessary.

(a) If A and B are positive constants, then the function $f(x) = \log(|Ax + B|)$ has a vertical asymptote at $x = -B/A$.

True False

(b) If an exponential function of t, in years, has decreased to 60% of the original value in two years, in four years it will decrease to 30% of the original value.

True False

(c) If $h(x) = 1.3(0.5)^x$ then the derivative, h', is decreasing for all x.

True False

(d) The functions $\sin(e^x)$ and $e^{sin(x)}$ are inverses of each other.

True False

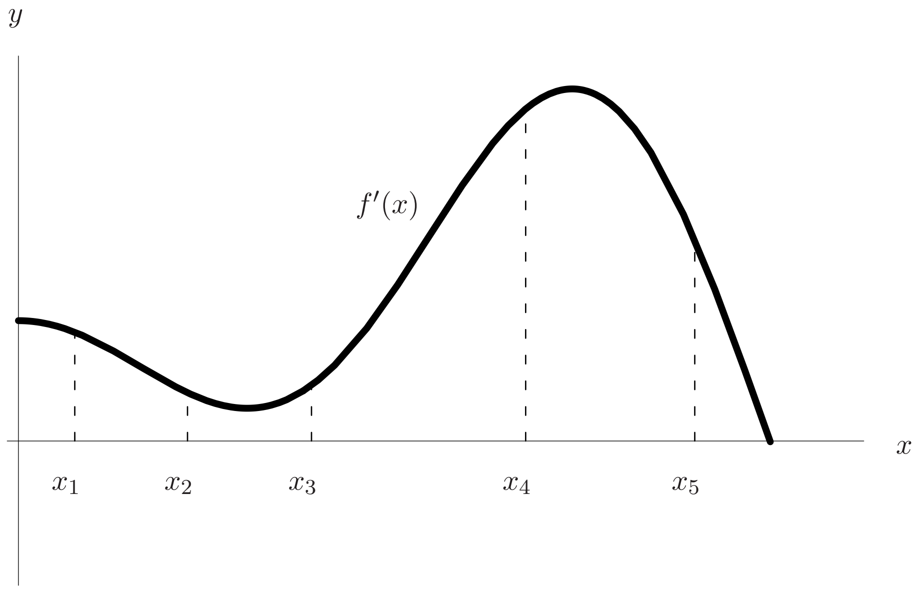
(e) If w is a continuous function for all x, then $\lim_{h\to 0} \frac{w(x+h) - w(x)}{h}$ exists for all x.

True False

(f) If $f''(x) > 0$ on the interval $[a, b]$, then the average rate of change of $f(x)$ on the interval $[a, b]$ is greater than $f'(x)$ for all $a < x < b$.

True False

2. (12 points) The graph of the *derivative* function, f', is given below. List all of the marked x-values, if any, from the figure for which the following statements are true. If no marked x-values apply, write "none."



(a) The value of $f(x)$ is greatest ________________________

(b) $f''(x) < 0$ ________________________

(c) f is decreasing ________________________

(d) Slope of f is positive ________________________

(e) The graph of f is concave up ________________________

3. (4 points) This exam will be graded out of 100 points. There are approximately 2000 students taking the exam. When the test has has been graded, there will be a function assigning to each student a score on the exam. Will this function be invertible? Why or why not?

4. (12 points) The cost of gasoline has risen dramatically in the last six months. At the beginning of March, the cost of gasoline was \$2.10 per gallon, but at the beginning of September, the cost was \$3.00 per gallon.

(a) Suppose the cost of gasoline, C, measured in dollars per gallon is a linear function of time, t. Find a formula for cost of gasoline as a function t, in months, since the beginning of March.

(b) Suppose further that you drive 200 miles per month and that your car averages 27 miles per gallon. Use your formula from part (a) to calculate the price of gasoline at the beginning of December. Assuming that the cost stays the same throughout the month of December, calculate your gas cost for the month of December.

(c) Now suppose instead that C is an exponential function of time. Find a formula for the cost of gas as a function of t, in months since March.

(d) Use your formula from part (c) to calculate the cost of gasoline at the beginning of December. Assuming that this cost remains the same throughout December, use the milage information from part (b) to calculate your total gasoline cost for December.

5. (12 points) In Ann Arbor the earliest sunset is at 4 p.m. and the latest at 8 p.m. (ignoring daylight savings time).

(a) Determine a trigonometric function, f, as a function of t in days, where $f(t)$ gives the number of hours past midnight when sunset occurs. Assume that $t = 0$ represents the winter solstice (December 21) and ignore leap years. [Recall that winter solstice is the shortest day of each year.]

(b) Give a practical interpretation of $f(90)$ in the context of this problem.

(c) Interpret $f'(120) = 0.03$ in the context of this problem.

(d) Suppose $g(x) = cf(x + h) - k$ for positive constants c, h and k. Give the following for $g(x)$ (your answers may involve c, h and k):

(i) Amplitude

(ii) Midline

(iii) Period

6. (10 points)

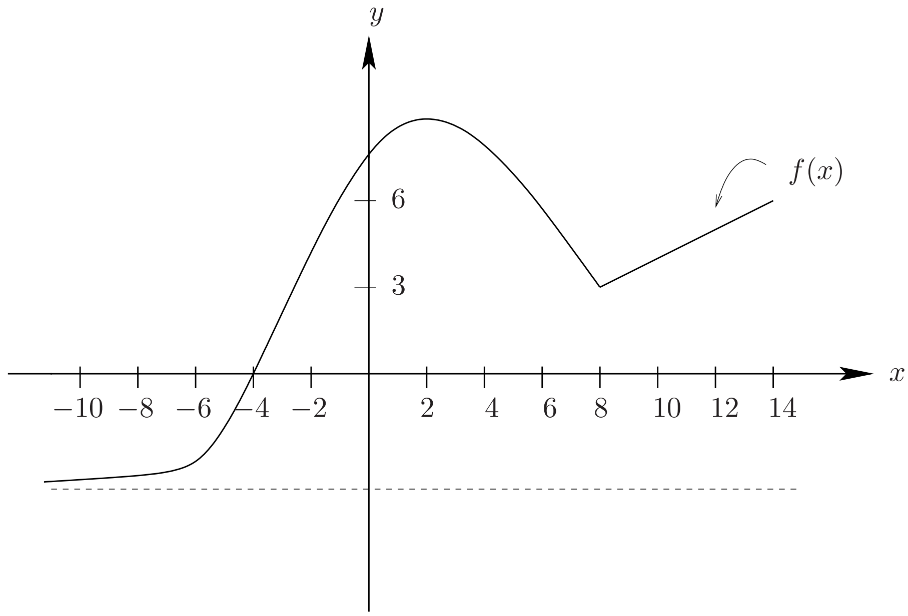
(a) Suppose that $h(x) = g(f(x))$. Fill in the missing values based on the information given in the table:

x	1	2	3
$f(x)$	-1	1	
$g(x)$	2/3	4/3	8/3
$h(x)$	1/6		8/3

(b) Which, if any, of f, g and h could be linear functions? Show evidence for your choice(s).

(c) Which, if any, of f, g and h could be exponential functions? Show evidence for your choice(s).

7. (12 points) The graph of a function f is given below.



(a) On the same set of axes, draw a graph of the derivative, $f'(x)$.

(b) Determine $f''(12)$.

(c) Describe in words what the expression $\dfrac{f(-2) - f(4)}{-6}$ represents graphically.

(d) Write the following slopes in increasing order:

$$\frac{f(2)}{2} \qquad \frac{f(14) - f(8)}{14 - 8} \qquad \frac{f(4)}{4}$$

8. (12 points) The potential energy E, in joules, of an object above the Earth's surface is a function of the distance, h, in meters, of the object from the surface of the Earth. That is, $E = f(h)$.

(a) In the context of this problem, explain the meaning of $f(20) = 1000$?

(b) In the context of this problem, explain the meaning of $f'(9) = 50$?

(c) In the context of this problem, explain the meaning of $f^{-1}(150) = 3$?

(d) In the context of this problem, explain the meaning of $(f^{-1})'(400) = \frac{1}{50}$?

9. (5 points) Write the **limit definition** for the derivative of $e^{\sin(x)}$ with respect to x. (No need to simplify or to attempt to find the limit.)

10. (9 points) Suppose

$$f(x) = \begin{cases} e^{\sin(x)} & x < \frac{\pi}{2} \\ kx & x \geq \frac{\pi}{2} \end{cases}$$

where k is some constant.

(a) If f is continuous, what is the value of k?

(b) Compute the average rate of change of f between $x = 1.5$ and $x = \frac{\pi}{2}$.

(c) Compute the average rate of change of f between $x = 1.57$ and $x = \frac{\pi}{2}$.

(d) Do you think f is differentiable at $x = \frac{\pi}{2}$? Explain your answer. [Your work from parts (a) - (c) may be useful here.]