MATH 115 — SECOND MIDTERM EXAM

November 22, 2005

NAME: ____

INSTRUCTOR: ____________________ SECTION NO: ________

1. Do not open this exam until you are told to begin.
2. This exam has 10 pages including this cover. There are 10 questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator. You are also allowed 2 sides of a 3 by 5 note card.
7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to make clear how you arrived at your solution.
8. Please turn **off** all cell phones and other sound devices, and remove all headphones.

PROBLEM	POINTS	SCORE
1	16	
2	9	
3	12	
4	10	
5	10	
6	6	
7	6	
8	10	
9	9	
10	12	
TOTAL	100	

1. (16 points) Use the information given below to answer the following questions. Show work where appropriate.

x	0	1	2	3	4
$f(x)$	0.5	2	2.5	0	-4
$f'(x)$	1.5	0.5	-1	-3	-3.5

(a) If $g(x) = Ax^2$ for some constant A, find $h'(2)$ where $h(x) = g(x) + f(2x)$. Your answer may involve the constant A.

(b) Suppose $k(x) = 4^{f(x)}$. Find $k'(1)$.

(c) Suppose $l(x)$ is a linear function of x, $l(4) = 0$, and $l'(4) < f'(4)$. Which of the following is true about $l(x)$? (Circle all that apply; you need not justify your answer):

(i) $l(x) > 0$ for $x > 4$.

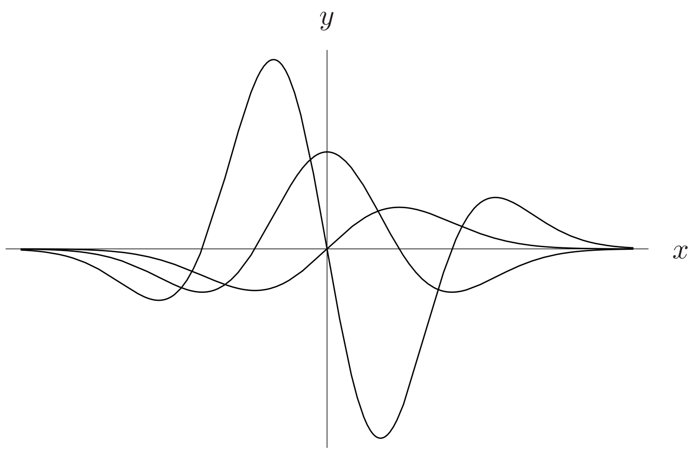
(ii) $l(x) < 0$ for $x > 4$.

(iii) $l(x)$ is increasing for all x.

(iv) $l(x)$ is decreasing for all x.

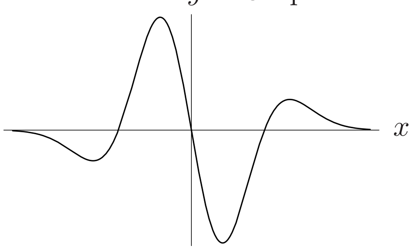
(d) Suppose $j(x)$ is an exponential function and that $j(0) = 1$. Let $h(x) = j(x)f(x)$. If $h'(0) = 7$, find a formula for $j(x)$.

2. (9 points) In the figure below, are the graphs of three functions, f, f', and f''. In the smaller figures, each graph is shown alone. To the right of the lower graphs give a clear explanation of how you determined which graph is f, which is f', and which is f''.

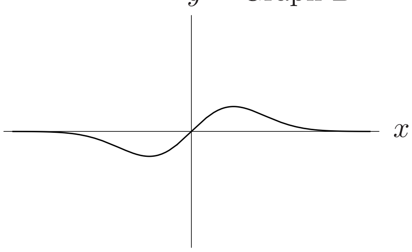


Explanation:

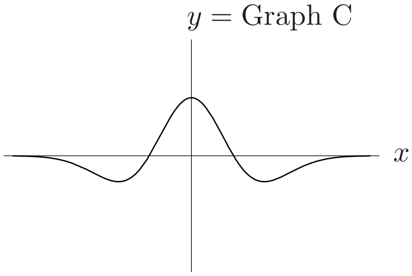
$y =$ Graph A



$y =$ Graph B



$y =$ Graph C



The graph of f corresponds to Graph____________________

The graph of f' corresponds to Graph____________________

The graph of f'' corresponds to Graph____________________

3. (12 points) For the function $f(x) = ax^4 - 3x^3$ with constant $a > 0$, use the techniques of calculus to answer the following. Show your work and proper justification for your answers.

(a) Determine all critical points of f. Classify each as a local maximum, a local minimum, or neither.

(b) Determine any global maxima or minima (if any).

(c) Determine all (if any) inflection points.

4. (10 points) Every year pesticides used on adjacent agricultural land drain off into Lake Michigan. Eventually, scientists predict that the lake will become saturated with pesticides. As a result, the amount of pesticides in the lake $P(t)$ (in parts per million) is given as a function of time, t, in years since 2000, by

$$P(t) = a(1 - e^{-kt}) + b$$

where a, b and k are positive constants. Assume the saturation level of the lake for pesticides is 50 parts per million.

(a) If in the year 2000 the pesticide level of Lake Michigan was 5 parts per million, find a and b.

(b) Find k if the pesticide level was increasing at a rate of 3 parts per million per year in the year 2000.

(c) When the pesticide level reaches 30 parts per million, fish from the lake cannot be consumed by humans. In what year will the pesticide level in the lake reach 30 parts per million?

5. (10 points)

(a) If the graph of

$$\frac{a}{y} + x^2 + b\ln y = 6$$

goes through the point $(2, 1)$ for some implicitly defined y, find a.

(b) Suppose $g(x) = -4x + 9$ is the equation of the tangent to the curve defined above at the point $(2, 1)$. Find b.

6. (6 points) Consider the function $f(x) = 3xe^{ax} + x^2$, where a is a constant. If the error in the linear approximation to $f(x)$ near $x = 0$ is 0.02 when $x = 0.1$, what is a? Show your work.

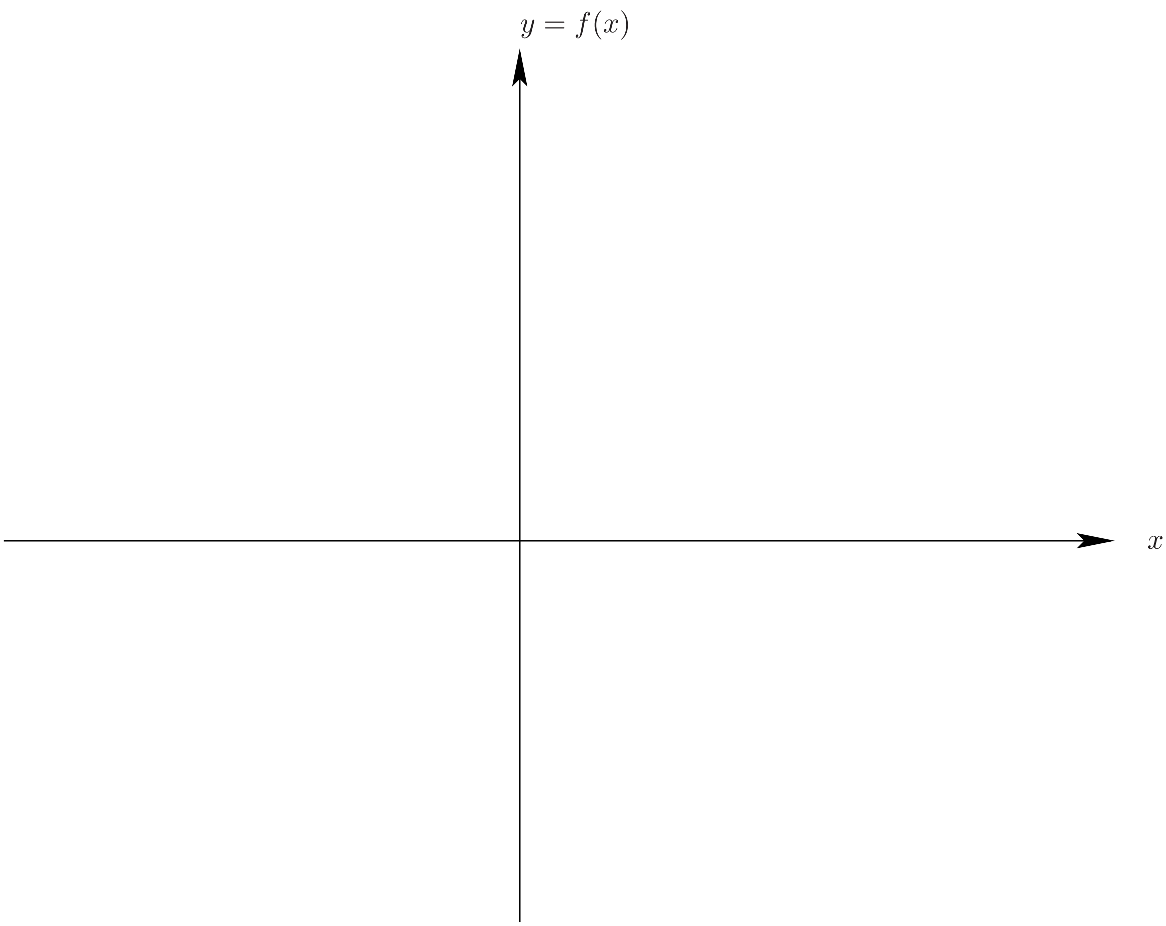
7. (6 points) The kinetic energy, K in Joules, of a particle in motion is a function of its fixed mass, M in kg, and its velocity, v, in $\frac{m}{s}$, and is given by:

$$K = \frac{1}{2}Mv^2.$$

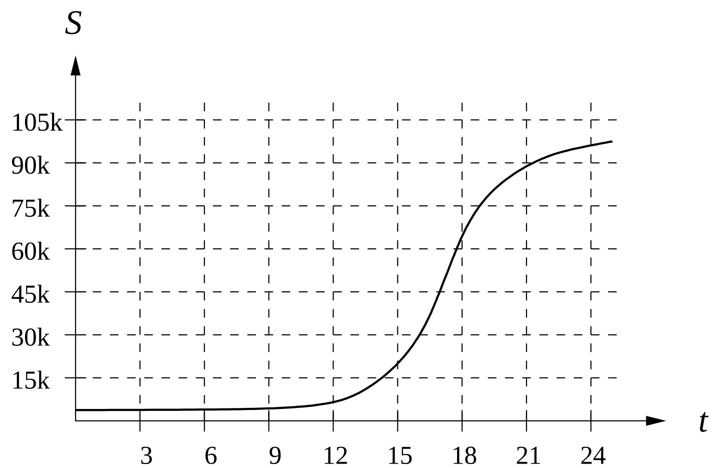
For an object with a mass of 2 kg, how fast is its kinetic energy increasing when it is traveling $3\frac{m}{s}$ and accelerating at a rate of $10\frac{m}{s^2}$?

8. (10 points) On the axes below, sketch a graph of a single, continuous, twice differentiable function f with all of the following properties. Be sure to clearly label your axes.

- $f(0) = 0$ and $\lim_{x\to\infty} f(x) = 4$
- $f'(x) = 0$ for $x = -2, 3$
- $f'(x) \geq 0$ for $-\infty < x < 3$
- $f'(x) < 0$ for $x > 3$
- $f''(x) = 0$ for $x = -2, 1, 5$
- $f''(x) > 0$ for $-2 < x < 1$
- $f''(x) < 0$ for $-\infty < x < -2$ and $1 < x < 5$



9. (9 points) Suppose that the starting salary, $S(t)$, at U-M for someone in the lucrative mathematics exam-formatting and writing business is given by the following figure. The units of S are given as tens of thousands of dollars, and S is a function of t, the number of years of education attained by the exam-formatter and writer. Suppose that we consider value $V(t)$ of an education to be the starting salary per year spent in school.



(a) How is $V(t)$ related to $S(t)$?

(b) How is $V(t)$ represented on the graph of $S(t)$ given?

(c) For $0 < t \leq 24$ what is the maximum of $V(t)$? According to the graph, how many years of education are the best value as far as starting salary is concerned? Explain.

10. (12 points) You are in charge of ticket sales for the U-M/Ohio State football game next year. Fans can buy pre-season tickets prior to September 1, 2006 for \$22.50 each. After September 1st, the price will be \$25 per ticket. The \$25 tickets are called term tickets. It turns out that pre-season ticket sales are a good predictor of term ticket sales, though the relationship is somewhat complicated. The number of term tickets sold, $T(x)$ (in thousands), is a function of the number of pre-season tickets sold, x (in thousands), and is given by:

$$T(x) = -0.02x^2 + 1.9x + 8.$$

Assume that the maximum capacity of the stadium is 115,000. What number of pre-season and term tickets should be sold to maximize revenue? Be sure to completely justify your answers–using techniques of calculus–(i.e., merely a graph or table will not suffice).