Math 115 – Second Midterm Exam

November 14, 2006

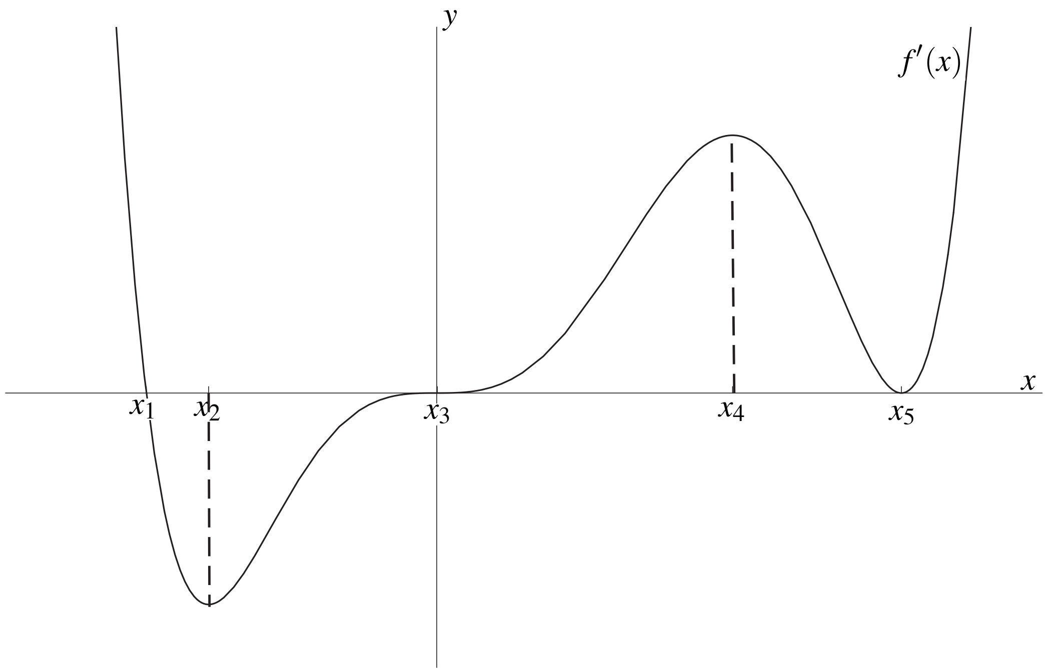
Name: ______________________________

Instructor: ______________________ Section Number: ______________

1. **Do not open this exam until you are told to begin.**
2. This exam has 8 pages including this cover. There are 7 questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator. You are also allowed two sides of a 3 by 5 notecard.
7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to show how you arrived at your solution.
8. Please turn off all cell phones and pagers and remove all headphones.

Problem	Points	Score
1	12	
2	20	
3	18	
4	12	
5	12	
6	14	
7	12	
Total	100	

1. (12 points) The graph below is a plot of $f'(x)$ (the *derivative* of f). Use the graph to answer questions about the function f.



(a) What are the critical points of f? ____________________

(b) For what value(s) of x does f have a local maximum? ____________________

(c) For what value(s) of x does f have a local minimum? ____________________

(d) What are the inflection points of f? ____________________

(e) On what interval(s) is f concave up? ____________________

(f) If f is a polynomial, what is the minimal degree of f? ____________________

2. (20 points) Suppose f, g, and h are all differentiable functions of x , $f(x)$ and $g(x)$ are positive for all x, and that a, and b are positive constants. Your answers below will be in terms of f, g, h (and/or their derivatives) and perhaps the constants a or b.

   (a) Find $\frac{dy}{dx}$ if $y = f(2) + \ln(f(x^2))$.

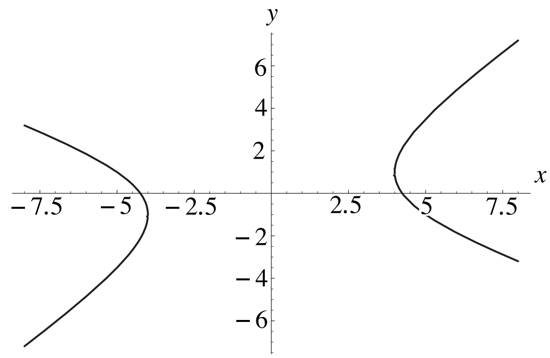
   (b) Find $\frac{dy}{dx}$ if $y = f(x^a + 2x) + 2^{g(x)}$.

   (c) Find $\frac{dy}{dx}$ if $y = \frac{h(bx)}{\cos(x) + 2}$.

   (d) If $f'(x) = ag(x)$ and $g'(x) = -af(x)$, when is $y = f(x)g(x)$ increasing? [Refer to the instructions above for conditions on f, g and a.] Justify your answer.

3. (18 points) Below is a graph of the curve implicitly defined by the equation

$$2y^2 - xy - x^2 = -18.$$



(a) Find a formula for $\dfrac{dy}{dx}$ as a function of both x and y.

(b) Find the value of $\dfrac{dy}{dx}$ at the point $(5, -1)$.

(c) Find any points (x_0, y_0) where $\dfrac{dy}{dx}$ is undefined, or give justification why no such points exist.

(d) Find any points (x_0, y_0) where $\dfrac{dy}{dx} = 0$, or give justification why no such points exist.

4. (12 points) Ellen and Renzo ran the Detroit marathon last weekend. The distance Ellen traveled (in meters) is given by $E(t)$ where t is time measured in seconds since the start of the race. Similarly, the distance in meters Renzo traveled is given by the function $R(t)$. For x measured in meters let $F(x) = R(E^{-1}(x))$. Assume that Ellen moves forward throughout the race–she does not even take a rest!

   (a) What is the practical interpretation of $F(50)$.

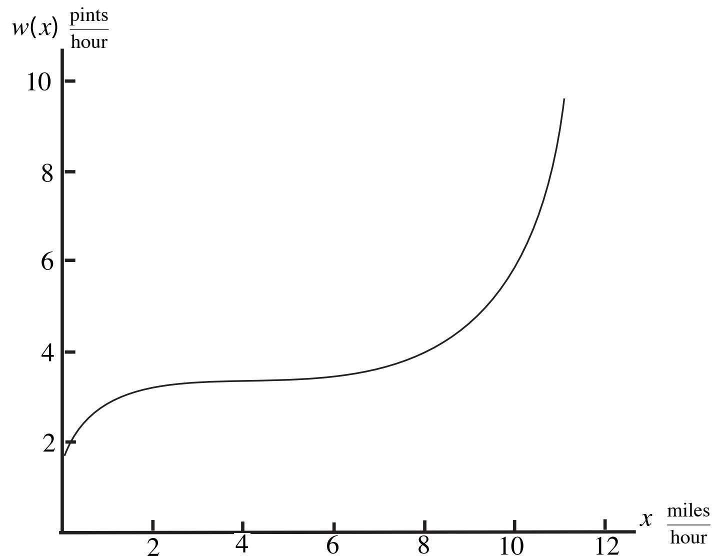
   (b) After the initial blast of speed from her start, Ellen ran at a constant rate of 5 meters per second for $2 < t < 10$, and she had run a distance of 39 meters after 7 seconds. Renzo wore a device that tracked the distance he had run at one second intervals. The data he collected is summarized in the table below.

t	0	1	2	3	4	5	6	7	8	9	10
$R(t)$	0	10	16	22	28	34	40	46	52	58	64

   Use any of the information above to approximate $F'(39)$.

   (c) Give a practical interpretation of $F'(39)$.

5. (12 points) Running a marathon takes a lot of energy. In order to keep up her energy level, Ellen drinks WolverineAid. Let $W(x)$ represent the number of pints of WolverineAid that Ellen must consume per hour when running at a rate of x miles per hour. The graph of $W(x)$ is given below.



(a) Let $C(x)$ represent Ellen's consumption of WolverineAid in pints per mile. How is $C(x)$ related to $W(x)$?

(b) Use calculus to show that $C(x)$ has a critical point at $x = x_0$ when $W'(x_0) = C(x_0)$. (Show your work.)

(c) From the graph, approximate the pace that Ellen should run in order to get the most efficient use of the WolverineAid. Explain your answer.

6. (14 points) Marathons are expensive. The city must use extra police for traffic control, set up aid stations, etc. However, for each runner in the race, the mayor estimates that the city will take in \$150 in revenue from entry fees and the expected items that the runners will buy before, after, and during the race.

   (a) The city's cost, in thousands of dollars, for the marathon is given by $C(n) = (n-10)^3 + 1000$, where n is the number, in *thousands*, of runners in the race. What is the Marginal Cost for 5000 runners?

   (b) If the city has 5000 entrants for the race, would they want to increase or decrease the number of entrants (or stay at 5000). Explain.

   (c) What number of runners will maximize the city's profit? [You may use your calculator for portions of this problem, but be sure to justify your answer.]

7. (12 points) The flux F, in millilitres per second, measures how fast blood flows along a blood vessel. Poiseuille's Law states that the flux is proportional to the fourth power of the radius, R, of the blood vessel, measured in millimeters. In other words $F = kR^4$ for some positive constant k.

   (a) Find a linear approximation for F as a function of R near $R = 0.5$. (Leave your answer in terms of k).

   (b) A partially clogged artery can be expanded by an operation called an angioplasty, which widens the artery to increase the flow of blood. If the initial radius of the artery was 0.5mm, use your approximation from part (a) to approximate the flux when the radius is increased by 0.1mm.

   (c) Is the answer found in part (b) an under- or over-approximation? Justify your answer.