Math 115 – Final Exam

December 18, 2006

Name: ____________________

Instructor: ____________________ Section Number: ____________

1. **Do not open this exam until you are told to begin.**
2. This exam has 8 pages including this cover. There are 7 questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator. You are also allowed two sides of a 3 by 5 notecard.
7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to show how you arrived at your solution.
8. Please turn off all cell phones and pagers and remove all headphones.

Problem	Points	Score
1	12	
2	24	
3	12	
4	10	
5	10	
6	16	
7	16	
Total	100	

1. (3 points each) In each of the following, circle **one** of the answers (A)-(E). No explanation necessary.

   (a) If f is differentiable for all x and has a local maximum at $x = 3$, then which of the following **must** be true?

   I. $f'(3) = 0$
   II. $f''(3) < 0$
   III. f is continuous at $x = 3$

   (A) I only   (B) II only   (C) I and II only

   (D) I and III only   (E) I, II, and III

   (b) If f and g are differentiable, $h(x) = f(x) - g(x)$, and $h(x)$ has a local maximum value at $x = 3$, then

   (A) $f'(x) > g'(x)$ (B)$f'(3) = g'(3)$ (C) $f'(3) < g'(3)$

   (D) $f(x)$ has a local maximum value at $x = 3$

   (E) $g(x)$ has a local minimum value at $x = 3$

   (c) Let $f(x) = \dfrac{\sin(x)}{e^x}$ for $x > 0$. When the minimum value of $f(x)$ occurs, then

   (A) $\sin(x) = 0$ (B) $\cos(x) = 0$ (C) $\cos(x) = \sin(x)$

   (D) $\cos(x) = -\sin(x)$ (E) $f(x)$ does not have any extreme values on the interval $[0, \infty)$

   (d) The graph of $y = x + \dfrac{1}{x}$ is both increasing and concave down on the interval

   (A) $(-\infty, -1)$ (B) $(-1, 0)$ (C) $(0, 1)$

   (D) $(1, \infty)$ (E) never

2. (4 points each) Suppose that f, g and h are continuous and differentiable functions such that $f'(x) = g(x)$ and **ALL** of the following conditions are also true:

$$\int_0^5 f(x)dx = -2, \qquad \int_5^{10} g(x)dx = 2, \qquad \int_0^5 g(x)dx = 15,$$

$$f(0) = 7, \qquad h(x) = g(x-5)$$

For parts (a)-(f), find the numerical value indicated. If insufficient information is given to answer the question indicate "Insufficient information".

(a) $\displaystyle\int_0^5 f(0)g(x)dx =$

(b) $f(10) =$

(c) $\displaystyle\int_0^5 |f(x)|\, dx =$

(d) $\displaystyle\int_0^5 \left(3f(0) - \frac{g(x)}{5}\right) dx =$

(e) $\displaystyle\int_0^5 \frac{1}{g(x)} dx =$

(f) $\displaystyle\int_5^{10} h(x)dx =$

3. (12 points) Consider the family of cubics of the form

$$f(x) = ax^3 + bx + c$$

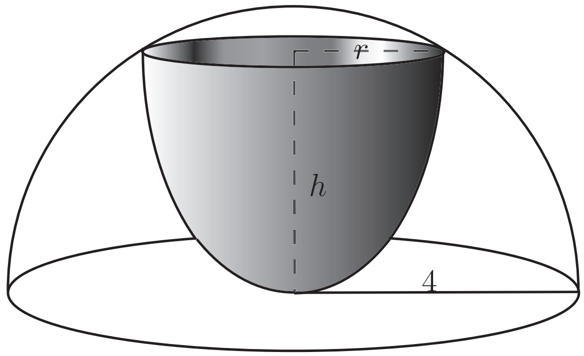
with a, b, and c non-zero constants.

(a) (2 points) Using the function $f(x) = ax^3 + bx + c$ as given above, write the **limit definition** of the derivative function, $f'(x)$. (No need to expand or simplify–just apply the definition to this function, using proper notation.)

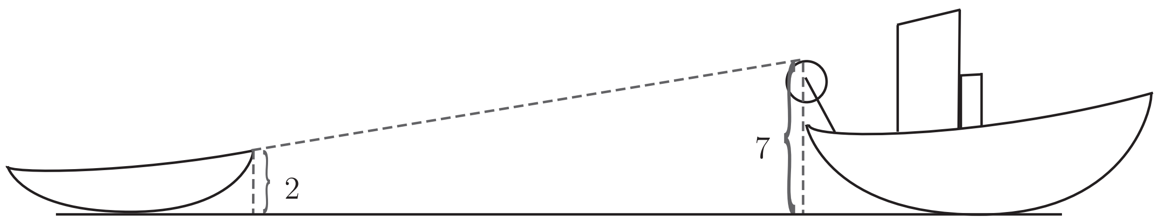
(b) (6 points) Under what conditions, if any, on a, b, and c will f have local extrema (i.e., maxima/minima)?

(c) (4 points) Under what conditions, if any, on a, b, and c will f have inflection point(s)?

4. (10 points) Suppose a paraboloid cup is inscribed in a hemisphere of radius 4 inches. The volume of the paraboloid is given by $\frac{1}{2}\pi r^2 h$. For what values of the parameters r and h is the volume of the cup maximized?



5. (10 points) A small boat has run out of gas. A cable is attached to the front of the boat 2 meters above the water. The other end of the cable is attached to a wheel of radius 0.5 meters sitting on the back of a tugboat. The top of the wheel is 7 meters above the water, and turns at a constant rate of 1 revolution per second. [See the figure below–not drawn to scale.]



(a) At what rate is the length of the cable between the two boats changing?

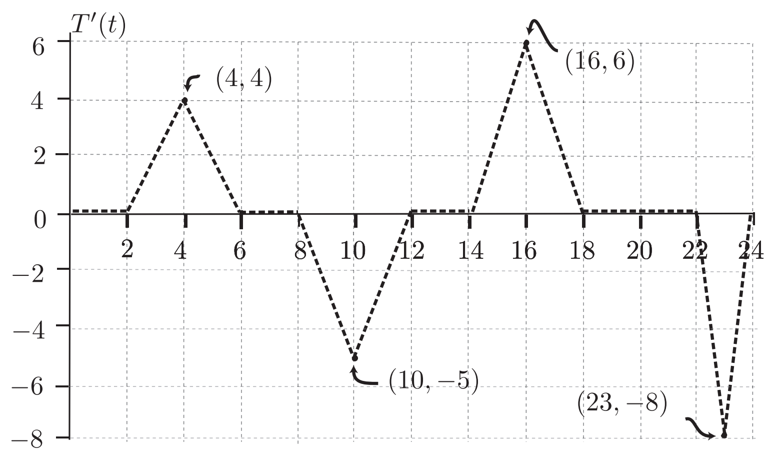
(b) How fast is the small boat being pulled forward when it is 10 meters away from the tugboat?

6. Suppose $H(c)$ gives the average temperature, in degrees, that can be maintained in Oscar's apartment during the month of December as a function of the cost of the heating bill, c, in dollars. In complete sentences, give a practical interpretation of the following:

   (a) (3 points) $H(50) = 65$

   (b) (3 points) $H'(50) = 2$

Suppose $T(t)$ gives the temperature in Oscar's apartment on December 18th in °F as a function of the time, t, in hours since 12:00 midnight. Below is a graph of $T'(t)$: (**NOTE: the graph is of $T'(t)$.**)

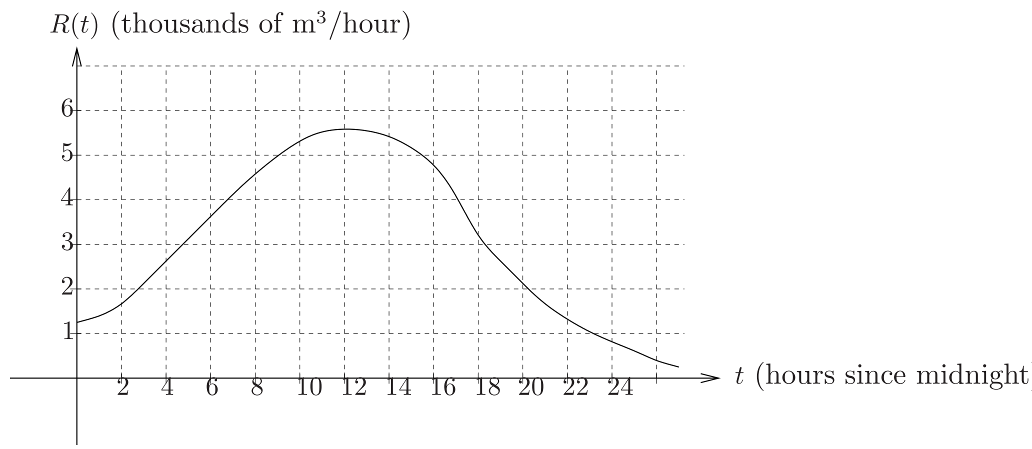


   (c) (6 points) When Oscar gets home from work at 6 pm the temperature in his apartment is 67 degrees. What was the temperature when he left for work at 8 am?

   (d) (4 points) If the temperature at 6 pm is 67 degrees, what is the minimum temperature in the apartment on December 18th?

7. Marie is already tired of winter. She is dreaming of her grandparents' farm and days rafting on the river near the farm. Not all days on the river are beautiful, though. One summer a storm dumped about a year's worth of rainfall on the area in a couple of days. A man-made lake held back by a dam near the farm rose as the swollen rivers rushed toward the lake.

The graph below gives the rate R, in thousands of cubic meters per hour, that water was entering the lake during that day as a function of t, in hours since midnight. The volume of the lake at midnight was 400,000 cubic meters. The maximum volume that can be held by the dam is 460,000 cubic meters. Due to an oversight, the floodgates of the dam were kept closed until 6:00 a.m when they were opened to full capacity. The gates allowed water to leave the lake at a constant rate of 2000 cubic meters per hour.



(a) (4 points) Approximate the volume of the lake when the floodgates were opened. Show your reasoning.

(b) (4 points) When did the lake reach its highest volume? Explain.

(c) (5 points) Approximately what was the highest volume of the lake on that day? Explain.

(d) (3 points) At what rate was the volume of the lake changing at 6:00 pm?.