

# MATH 115 –FIRST MIDTERM EXAM

October 9, 2007

NAME: \_\_\_\_\_

INSTRUCTOR: \_\_\_\_\_ SECTION NUMBER: \_\_\_\_\_

---

1. **Do not open this exam until you are told to begin.**
2. This exam has 9 pages including this cover. There are 9 questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator. You are also allowed two sides of a 3 by 5 notecard.
7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to show how you arrived at your solution.
8. Please turn off all cell phones and pagers and remove all headphones.

PROBLEM	POINTS	SCORE
1	18	
2	14	
3	13	
4	7	
5	8	
6	12	
7	6	
8	14	
9	8	
TOTAL	100	

1. (3 points each. No partial credit.) The questions on this page are True or False. They do not require an explanation. For each question, circle your choice for the correct answer. Only answer True when the statement is **ALWAYS** True.

(a) The function  $f(x) = \frac{e^x}{x^2 - 1}$  is continuous on  $[2, 5]$ .

True      False

(b) Suppose  $g$  is a differentiable function on  $(-1, 1)$  with  $g(1) < 0$  and  $g'(x) > 0$  for  $x$  in  $(-1, 1)$ , then  $g(x)$  has a zero on the interval  $[-1, 1]$ .

True      False

(c) If  $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x)$  then  $f$  is continuous at  $x = 0$ .

True      False

(d) If  $x > 0$  and  $e^{xy-2} = x^2$ , then  $y = \frac{2}{x}(1 + \ln x)$ .

True      False

(e) A function that is continuous on  $[a, b]$  is always differentiable on  $[a, b]$ .

True      False

(f) If  $f'(a) = 0$  and  $f'(x) > 0$  for  $x < a$ , then  $f''(x) > 0$  for  $x < a$ .

True      False



3. Dr. Charles Keeling began measuring carbon dioxide in the atmosphere on a continuous basis in 1958. At that time, Dr. Keeling found that the mean concentration level was approximately 315 ppmv. Currently, the level is approximately 385 ppmv. <sup>2</sup>

(a) (4 points) Assuming that the mean concentration has been growing linearly from 1958 to 2007, find a formula for  $L(t)$ , the mean concentration level of CO<sub>2</sub>, with  $t$  in years since 1958.

(b) (5 points) If instead, the mean concentration has been growing at an exponential rate, find an exponential function,  $E(t)$ , to model the mean concentration level of CO<sub>2</sub> in the environment  $t$  years after 1958.

(c) Future CO<sub>2</sub> levels are expected to rise due to burning of fossil fuels and land-use changes. The rate of this increase will depend on uncertain economic, sociological, technological, and natural developments. The IPCC Special Report on Emissions Scenarios gives a wide range of CO<sub>2</sub> scenarios by the year 2100. Use your functions from parts (a) and (b) to predict the concentration of CO<sub>2</sub> in 2100.

(i) (2 points) Prediction if growth is linear:

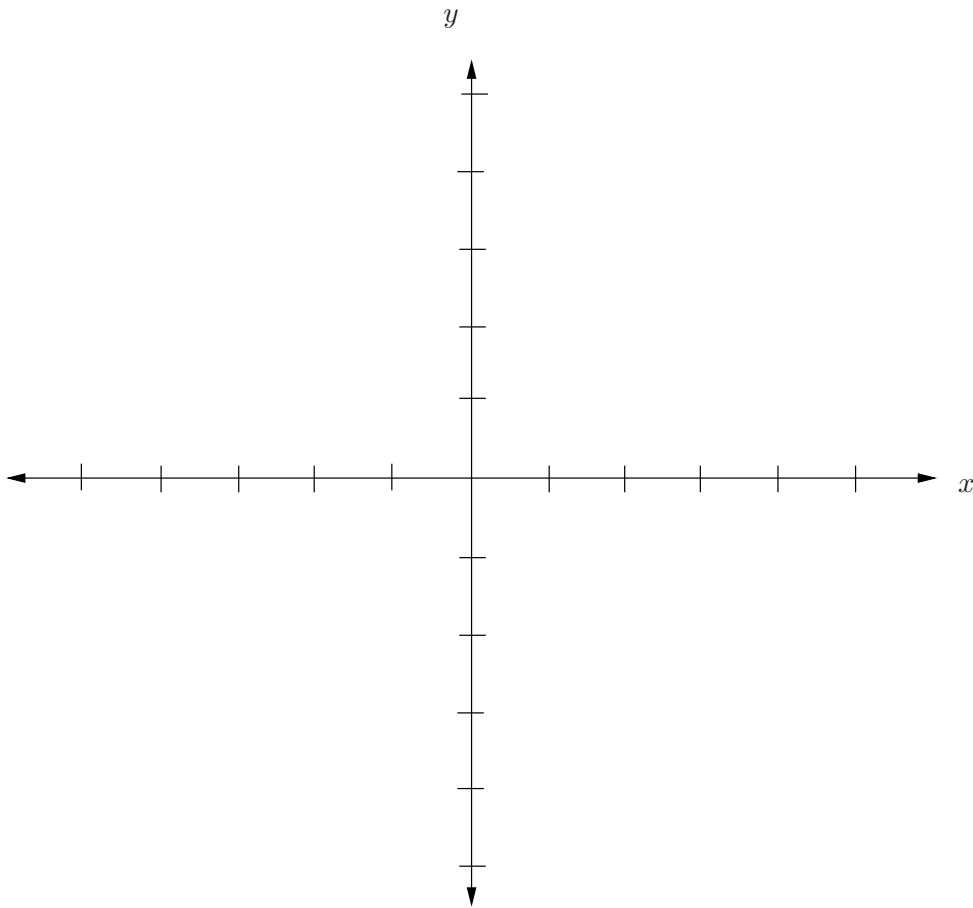
(ii) (2 points) Prediction if growth is exponential:

---

<sup>2</sup>See [http://earthguide.ucsd.edu/globalchange/keeling\\_curve/01.html](http://earthguide.ucsd.edu/globalchange/keeling_curve/01.html) and [http://en.wikipedia.org/wiki/Carbon\\_dioxide](http://en.wikipedia.org/wiki/Carbon_dioxide)

4. (7 points) On the axes below, sketch a graph of a single function,  $g$ , with all of the following properties.

- $g(-2) = 0$
- $g'(x) = -1$  for  $x < -2$
- $\lim_{x \rightarrow -2^-} g(x) = \lim_{x \rightarrow -2^+} g(x)$
- $g''(x) > 0$  for  $-2 < x < 0$
- $g'(1) = 0$
- $\lim_{x \rightarrow 3^-} g(x) = -4$
- $g(x) = 2$  for  $x \geq 3$



5. (a) (4 points) Let the function  $f$  be defined as follows:

$$f(x) = \begin{cases} 2^p (x - 1) & \text{for } x > 2 \\ x^2 & \text{for } 0 \leq x \leq 2 \\ \cos(x^2) + k & \text{for } x < 0 \end{cases}$$

Find the values of  $p$  and  $k$  so that  $f$  is a continuous function.

$$p = \underline{\hspace{2cm}}$$

$$k = \underline{\hspace{2cm}}$$

(b) (4 points) Using  $f(x)$  as determined in part (a) and  $g(x)$  given by:

$$g(x) = \begin{cases} \frac{x^3}{3} & \text{for } x \geq 3 \\ |x| & \text{for } x < 3 \end{cases}$$

find

(i)  $\lim_{x \rightarrow 3^+} f(x)g(x)$

(ii)  $\lim_{x \rightarrow 3^-} f(x)g(x)$

6. Adrian wants to conserve energy. She learned that as a nation we spend approximately one quarter of our electricity on lighting. In our homes, the amount of electricity used for lighting can be reduced by using fluorescent light bulbs. If Adrian's monthly electricity bill,  $L$  in dollars, is a function of the percent,  $p$ , of fluorescent bulbs in her home, give the practical interpretation of the each of following—*i.e.*, give the meaning of the expression or statement as you would explain it to a person who knows no mathematics.

(a) (4 points) In the context of this problem,  $L(0) = 100$  indicates...

(b) (4 points) In the context of this problem,  $L'(25) = -2$  indicates...

(c) (4 points) What does  $L^{-1}(75)$  stand for?

7. (6 points) In the wee hours of October 1st, the Michigan governor and state legislators came up with a new function called "Save Our State," denoted  $\text{SOS}(x)$ . The function is defined for all values and differentiable everywhere. (We all hope the slope is *positive!* ) Use the **limit** definition of the derivative to define  $f'(2)$  in terms of the function  $\text{SOS}$ , if

$$f(t) = \text{SOS}(2t^2).$$

8. David is living on the 37th floor of a fancy building. He wants to get rid of an ancient (very energy inefficient) refrigerator that was in the building before alterations were made to the apartment. The box will not fit through the new doors of the apartment, so the refrigerator must be pushed down a rather rickety ramp out the window. The ramp is 350 feet long. Below is a table showing the distance from the window along the ramp at given times:

time (seconds)	0	2	4	6	8	10	12	14	16
distance from window (feet)	0	3.9	18.6	43.1	79.4	122.5	174.6	240.1	313.6

Suppose  $s(t) = d$  is the distance from the window, in feet, as a function of time,  $t$ , in seconds.

- (a) (3 points) Compute the average velocity of the refrigerator over the time interval  $4 \leq t \leq 12$ .

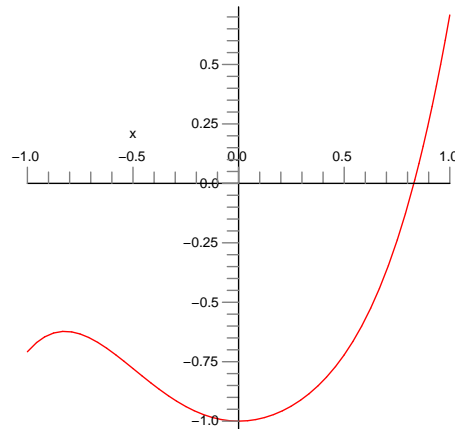
- (b) (4 points) Approximate the instantaneous velocity of the refrigerator when  $t = 8$  seconds.

- (c) (3 points) Approximately where will the refrigerator be after 18 seconds? Justify your answer.

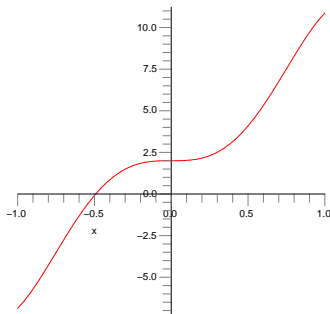
- (d) (4 points) Based upon the information in the table, does  $s$  appear to be concave up or concave down at  $t = 8$ ? Justify your answer.



9. (8 points) The graph of  $y = g(x)$  is given by the figure below.



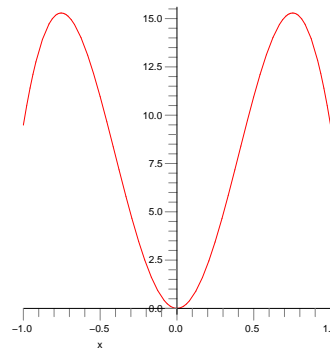
The graphs of the following functions include the graphs of  $g'$  and  $g''$  and two additional graphs. On the lines beneath the appropriate graphs, **clearly** identify the graphs of  $g'$  and  $g''$ . (No explanation necessary.) For each of the other graphs, give *any* feature of the graph (e.g., its behavior on an interval or at a point) which disqualifies that graph as a candidate for the derivative of  $g$ . (No need to list *all* reasons!)




---



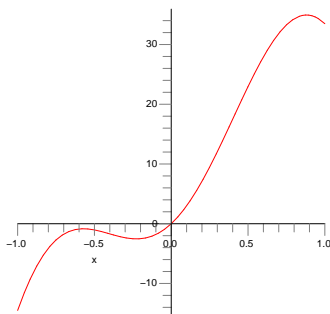
---




---



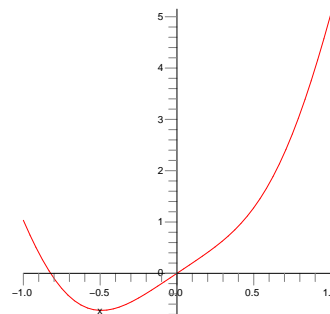
---




---



---




---



---