MATH 115 – SECOND MIDTERM EXAM

November 13, 2007

NAME: ______________________________

INSTRUCTOR: ____________________ SECTION NUMBER: ____________

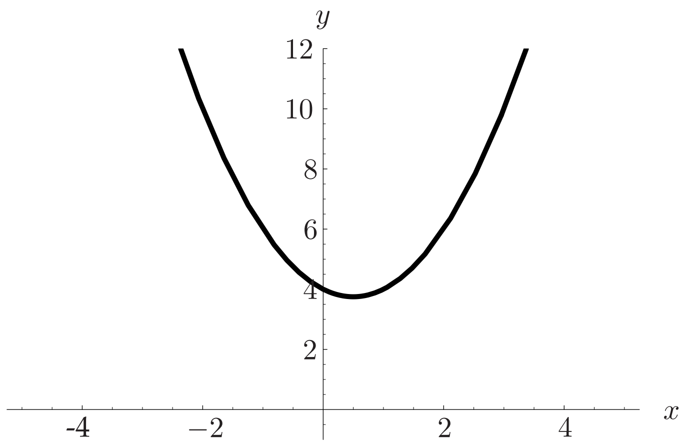
1. **Do not open this exam until you are told to begin.**
2. This exam has 8 pages including this cover. There are 8 questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator. You are also allowed two sides of a 3 by 5 notecard.
7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to show how you arrived at your solution.
8. Please turn off all cell phones and pagers and remove all headphones.

PROBLEM	POINTS	SCORE
1	10	
2	20	
3	14	
4	10	
5	12	
6	5	
7	12	
8	17	
TOTAL	100	

1. (10 points) Using the graph in the figure below to help you, find the equations of all lines through the origin and tangent to the parabola

$$y = x^2 - x + 4.$$

[Show all relevant work—graphical approximations are not sufficient. Please circle your answers.]



2. (20 points) Suppose f and g are differentiable functions with the following values:

$$f(0) = 3, \quad f'(0) = 4, \quad g(0) = -1, \quad \text{and} \quad g'(0) = 2.$$

Show your work on the following:

(a) Find $h'(0)$ given $h(x) = \dfrac{g(x)}{f(x)}$.

(b) i. Find $k'(0)$ given $k(x) = (g(x))^2 f(x)$.

ii. Determine the local linearization of $k(x)$ near $x = 0$, and use that to approximate $k(0.001)$.

(c) Find $m'(0)$ given $m(x) = \sin\left((f(x))^3\right)$.

3. (14 points) Find the values of the constants $a,\ b$ and c such that the function

$$f(x) = ax^2 + bx + c$$

"fits" the function

$$g(x) = -2\ln(x+1) + 0.5e^x + 1.5\sin(x)$$

near $x = 0$ in the sense that:

$$g(0) = f(0), \quad g'(0) = f'(0) \quad \text{and} \quad g''(0) = f''(0).$$

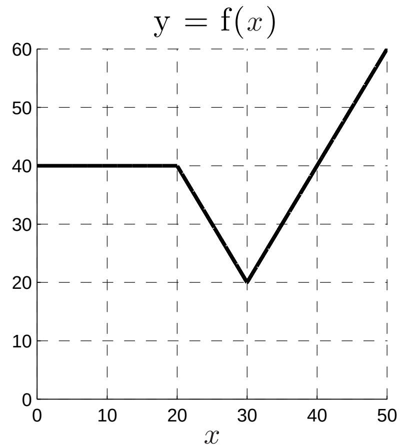
Show all work.

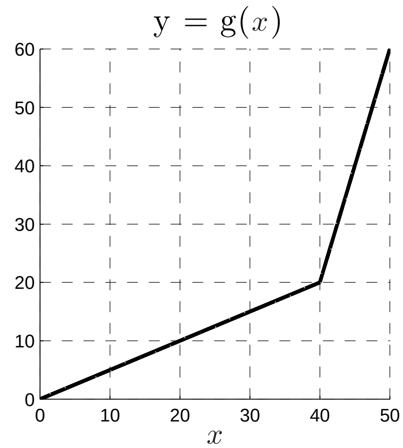
$a =$ ________________

$b =$ ________________

$c =$ ________________

4. (10 points) Consider the graphs of $f(x)$ and $g(x)$ below. Let $h(x) = f(g(x))$.

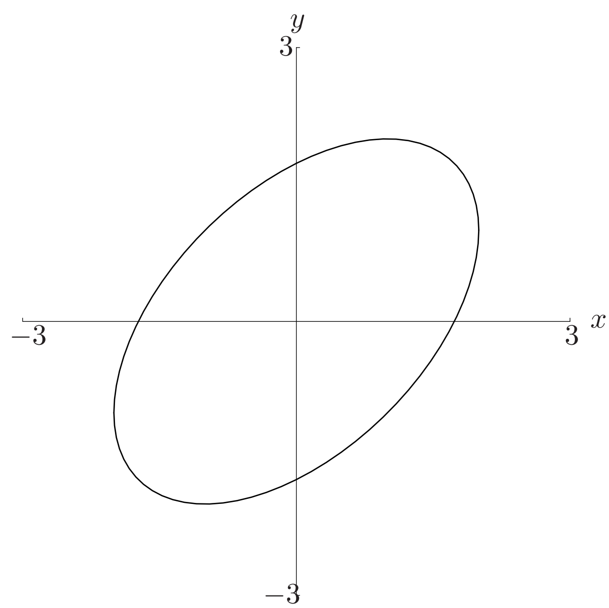




(a) Evaluate $h'(30)$ exactly. Show your work.

(b) Determine the range of values of x for which $h'(x) < 0$. Justify your answer.

5. (12 points) The graph of
$$x^2 - xy + y^2 = 3$$
is a "tilted" ellipse (see the figure below). Among all points (x, y) on this graph, find the points that have the largest and smallest values of y. [Hint: Look at the figure to consider the conditions that would be true for y to take on largest or smallest values.] Be sure to show all work in order to justify your answer (*i.e..* estimating points from a graph will not be sufficient).



Largest y value is associated with the point: ____________________

Smallest y value is associated with the point: ____________________

6. (5 points) Parasitoids are insects that lay eggs in, on, or close to other (host) insects. Their larvae then devour the host insect, resulting in the death of the host. The likelihood of escaping parasitism may depend on parasitoid density. One such model sets the probability, P, of escaping parasitism as:

$$P = f(D) = \left(1 + \frac{a\,D}{k}\right)^{-k}$$

where D is the parasitoid density and a and k are positive constants.

Determine whether the probability of escaping parasitism increases or decreases as parasitoid density increases. Justify your answer.

7. (12 points) Let $P = f(t)$ be the total amount, in trillions of barrels, of the world's reserves of petroleum in year t.

(a) What does the statement $f(2006) = 1.2$ tell you about the petroleum reserves?

(b) Evaluate and interpret $f^{-1}(1.2)$.

(c) What does the statement $f'(2006) = -0.003675$ tell you about the petroleum reserves?

(d) Evaluate and interpret $\left(f^{-1}\right)'(1.2)$.

8. (17 points)

   (a) A small company makes x hand-painted tiles daily at a cost of $C(x) = 125 + 30x + 2x^{3/2}$ dollars. What daily production level minimizes the *average* cost—*i.e.*, the cost per tile?

   (b) If the the company sells each tile for \$75, how many tiles should they make daily in order to maximize daily profit?