MATH 115 – SECOND MIDTERM

November 18, 2008

NAME: ____________________

INSTRUCTOR: ____________________ SECTION NUMBER: ____________________

---

1. **Do not open this exam until you are told to begin.**
2. This exam has 8 pages including this cover. There are 8 questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam. If you need extra room, you may use the back of a page but be *sure* to clearly indicate and label your work.
4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator. You are also allowed two sides of a 3 by 5 notecard.
7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to show how you arrived at your solution.
8. Please turn off all cell phones and pagers and remove all headphones.

PROBLEM	POINTS	SCORE
1	10	
2	12	
3	18	
4	8	
5	6	
6	16	
7	16	
8	14	
TOTAL	100	

1. For the following questions select true if the statement is *always* true, and false otherwise. Each question is worth 1 point.

   (a) If f is differentiable and $f'(p) = 0$ or $f'(p)$ is undefined, then $f(p)$ is either a local maximum or a local minimum.

   True False

   (b) For f a twice differentiable function, if f' is increasing, then f is concave up and increasing.

   True False

   (c) The global maximum of $f(x) = x^2$ on every closed interval is at one of the endpoints of the interval.

   True False

   (d) If $f(x)$ has an inverse function $g(x)$, then $g'(x) = 1/f'(x)$.

   True False

   (e) If a function is periodic with period c, then so is its derivative.

   True False

   (f) If $C(q)$ represents the cost of producing a quantity q of goods, then $C'(0)$ represents the fixed costs.

   True False

   (g) If a differentiable function $f(x)$ has a global maximum on the interval $0 \leq x \leq 10$ at $x = 0$, then $f'(x) \leq 0$ for $0 \leq x \leq 10$.

   True False

   (h) If $f(x)$ is differentiable and concave up, then $f'(a) < \frac{f(b)-f(a)}{b-a}$ for $a < b$.

   True False

   (i) If you zoom in with your calculator on the graph of $y = f(x)$ in a small interval around $x = 10$ and see a straight line, then the slope of that line equals the derivative $f'(10)$.

   True False

   (j) If $f'(x) \geq 0$ for all x, then $f(a) \leq f(b)$ whenever $a \leq b$.

   True False

2. In 1956, Marion Hubbert began a series of papers predicting that the United States' oil production would peak and then decline. Although he was criticized at the time, Hubbert's prediction was remarkably accurate. He modeled the annual oil production $P(t)$, in billions of barrels of oil, over time t, in years, as the *derivative* of the *logistic function* $Q(t)$ given below—*i.e.*, $Q'(t) = P(t)$. The function P is measured in years since the middle of 1910.

   The function $Q(t)$ is given by

$$Q(t) = \frac{Q_0}{1 + ae^{-bt}}\text{, where } a, b, Q_0 > 0. \tag{1}$$

   For your convenience, the first and second derivatives of $Q(t)$ are given as well:

$$Q'(t) = -\frac{Q_0}{(1 + ae^{-bt})^2}\left(-abe^{-bt}\right) = \frac{abQ_0e^{-bt}}{(1 + ae^{-bt})^2},$$

   and

$$Q''(t) = \frac{ab^2Q_0e^{-bt}}{(1 + ae^{-bt})^3}\left[ae^{-bt} - 1\right].$$

   (a) (2 points) Interpret, in the context of this problem, $P'(56)$.

   (b) (6 points) Determine the year of maximum annual production t_{max}. Your answer may involve all or some of the constants a, b, Q_0.

   (c) (2 points) Find the maximum annual production $P(t_{max})$. Again, your answer may involve all or some of the constants a, b, Q_0.

   (d) (2 points) In his 1962 paper, Hubbert studied the available data on oil production to date and concluded that $a = 46.8$, $b = 0.0687$, and $Q_0 = 170$ Bb (billion barrels). Using your results from part (b), when would Hubbert's curve predict the peak in US oil production? (The actual peak occurred in 1964.)

3. Use the information below to find an equation that best models the situation and most accurately fits the given data.

(a) i. (2 points) Suppose a pair of shoes at DSW costs $\$50$ after a 10% discount. Find a formula for $P(n)$, the price of the shoes after n discounts of 10%, where $n \geq 0$.

ii. (4 points) Find and interpret $P'(4)$ in the context of this problem.

(b) (6 points) Michigan's population (in millions) for the last three years as measured by the U.S. Census Bureau is given below.

Year	2005	2006	2007
Population	10.108	10.102	10.071

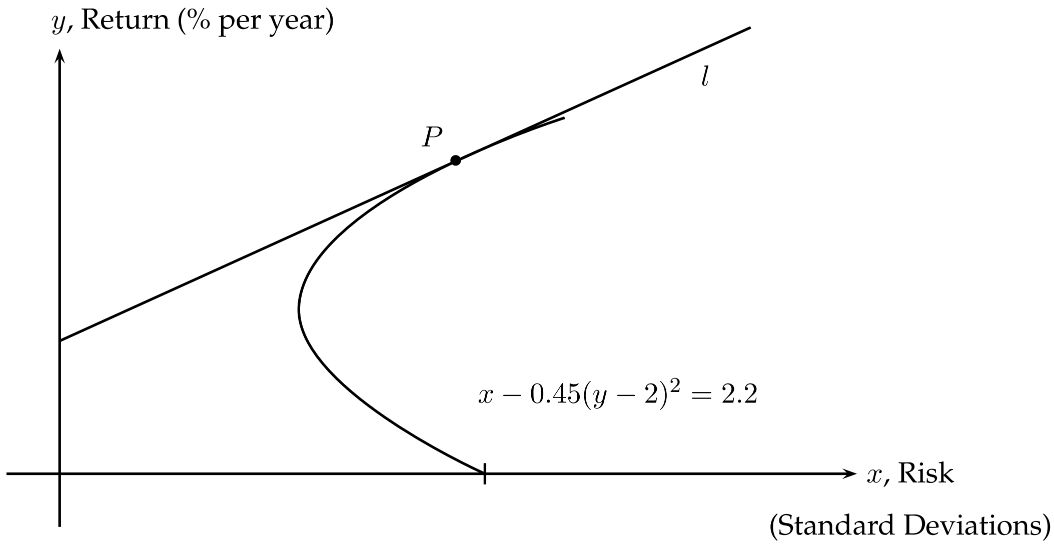
Find a formula to approximate the population of Michigan, $P(t)$, with t in years since 2005. Using this information, approximate the population of Michigan in 2008. Show your work.

(c) (6 points) The height $h(t)$ (in ft. above the ground) of a passenger on a ferris wheel (a circular fair ride) varies from a maximum of 50 ft. to a minimum of 2 ft. as a function of time t (in minutes). If the ferris wheel makes 0.1 revolutions/minute, and the passenger is initially at the top of the ride, find a formula for the vertical velocity of the passenger, $v(t)$.

4. (8 points) Determine a and b for the function of the form $y = f(t) = at^2 + b/t$, with a local minimum at (1,12).

5. (6 points) The circulation time of a mammal (that is, the average time it takes for all the blood in the body to circulate once and return to the heart) is proportional to the fourth root of the body mass of the mammal. The constant of proportionality is 17.40 if circulation time is in seconds and body mass is in kilograms. The body mass of a certain growing child is 45 kg and is increasing at a rate of 0.1 kg/month. What is the rate of change of the circulation time of the child?

6. In Modern Portfolio Theory, a client's portfolio is structured in a way that balances risk and return. For a certain type of portfolio, the risk, x, and return, y, are related by the equation $x - 0.45(y-2)^2 = 2.2$. This curve is shown in the graph below. The point P represents a particular portfolio of this type with a risk of 3.8 units. The tangent line, l, through point P is also shown.

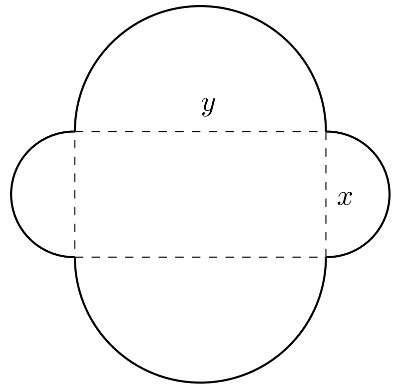


(a) (5 points) Using implicit differentiation, find dy/dx, and the coordinates of the point(s) where the slope is undefined.

(b) (8 points) The y-intercept of the tangent line for a given portfolio is called the *Risk Free Rate of Return*. Use your answer from (a) to find the Risk Free Rate of Return for this portfolio.

(c) (3 points) Now, estimate the return of an optimal portfolio having a risk of 4 units by using your information from part (b). Would this be an overestimate or an underestimate? Why?

7. The figure below is made of a rectangle and semi-circles.



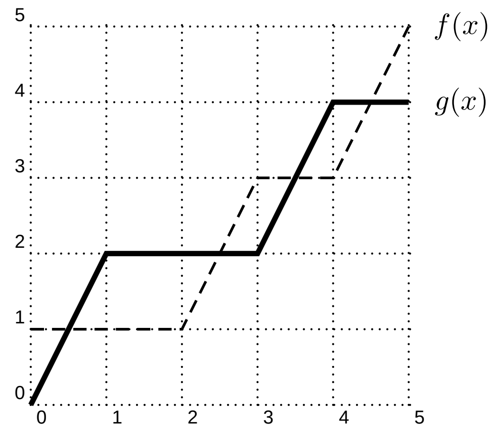
(a) (3 points) Find a formula for the enclosed area of the figure.

(b) (2 points) Find a formula for the perimeter of the figure.

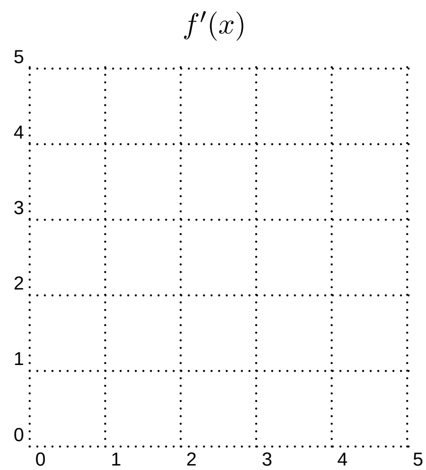
(c) (8 points) Find the values of x and y which will maximize the area if the perimeter is 100 meters.

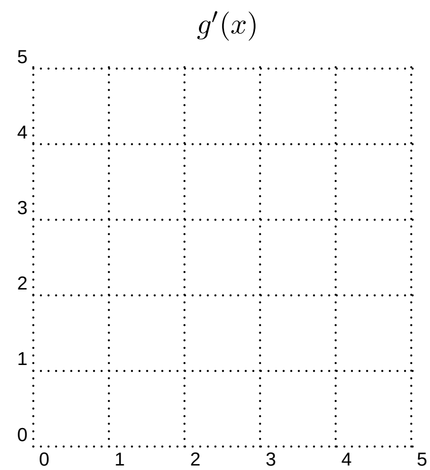
(d) (3 points) If the cost, in dollars, of the materials to build the enclosure is given by $C(x)$ where x is in meters, and the Marginal Cost at $x = 100$ is 25, what does this mean in the context of the problem?

8. (14 points) Use the functions $f(x)$ and $g(x)$ graphed below to answer the following questions:



(a) (3 points each) Graph $f'(x)$ and $g'(x)$.





(b) (2 points) Compute $h'(3)$ for $h(x) = f(g(x))$.

(c) (2 points) Define $r(x) = g(x) - f(x)$. For what x value(s) in $[0, 5]$ is $r(x)$ maximum?

(d) (2 point) Find $s'(2.5)$ for $s(x) = f(x)g(x)$.

(e) (2 points) Find $w'(2.5)$ for $w(x) = f(x)/g(x)$.