MATH 115 – FINAL EXAM

December 11, 2008

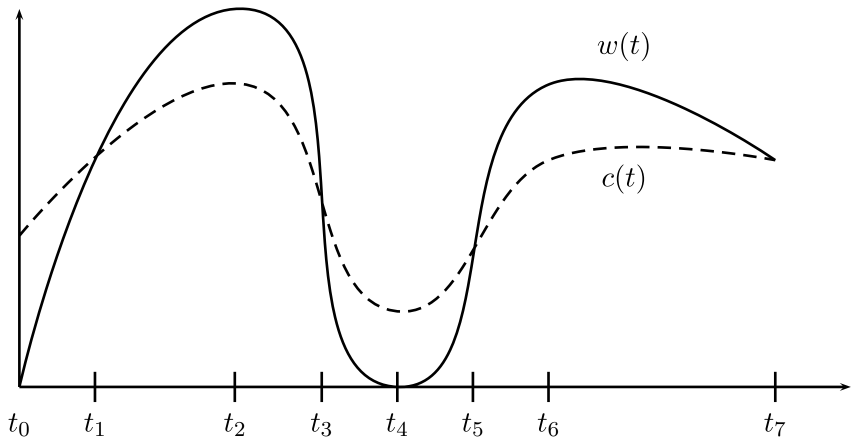
NAME: ______________________________

INSTRUCTOR: ______________________ SECTION NUMBER: ______________

1. **Do not open this exam until you are told to begin.**
2. This exam has 8 pages including this cover. This exam has 8 questions, and your score on the CCI constitutes an additional 9th problem, worth five points.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator. You are also allowed two sides of a 3 by 5 notecard.
7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to show how you arrived at your solution.
8. Please turn off all cell phones and pagers and remove all headphones.

PROBLEM	POINTS	SCORE
1	12	
2	10	
3	20	
4	11	
5	15	
6	7	
7	8	
8	12	
9	5	CCI score
TOTAL	100	

1. In the 17th century, a ship's navigator would estimate the distance the ship has traveled using readings of the ship's velocity, $v(t)$, in knots (nautical miles per hour). Suppose that between noon and 3:00 pm a certain galleon is traveling with the wind and against the ocean current, and that its velocity is given as the difference between the wind velocity $w(t)$ and the velocity of the ocean current $c(t)$, so that $v(t) = w(t) - c(t)$, where t is in hours since noon. Consider the wind and ocean velocities for various times between noon and 3:00 p.m., given by the graphs below:



(a) (1 point) Using *integral notation* write an expression giving the distance the ship traveled from noon to 3:00 pm. Give units.

(b) (1 point) Using *integral notation* write an expression giving the average velocity of the ship between noon and 3:00 pm. Give units.

(c) (2 points) For what intervals was the ship's velocity positive?

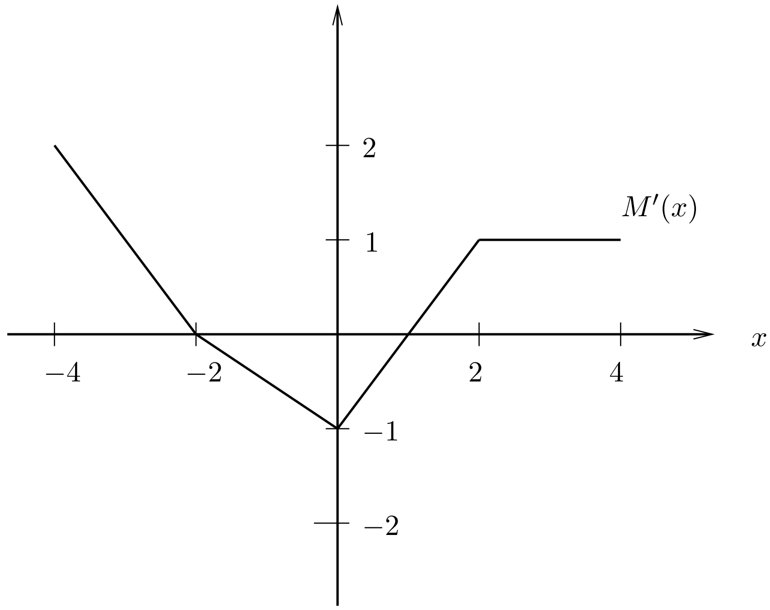
(d) (2 points) For what t values was the ship not moving towards its destination?

(e) (2 points) For what intervals was the ship's velocity increasing?

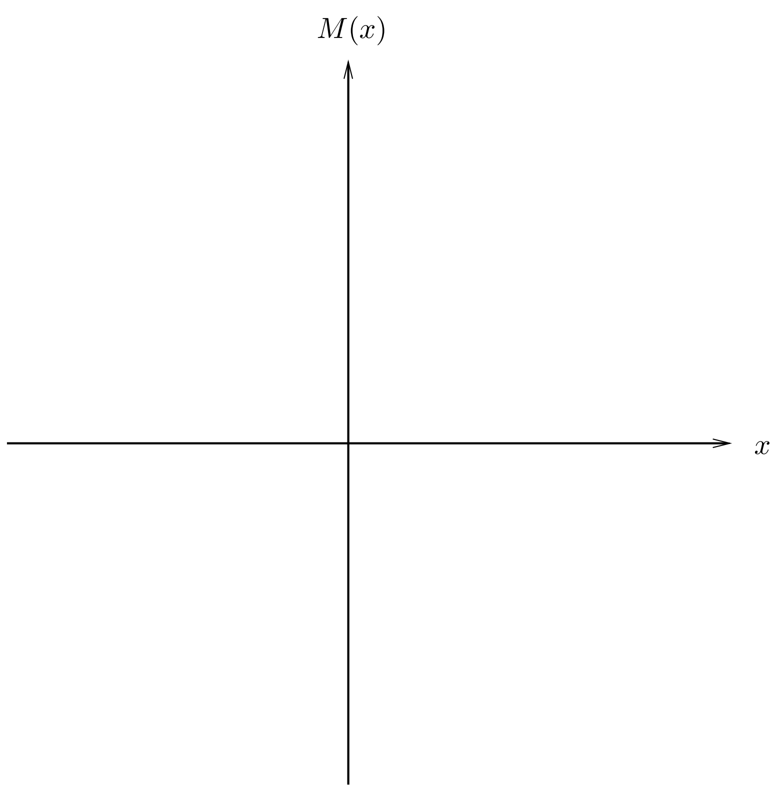
(f) (4 points) Please circle each integral which is positive and underline each integral which is negative.

$$\int_{t_1}^{t_3} v(t)dt \qquad \int_{t_5}^{t_7} v(t)dt \qquad \int_{t_0}^{t_7} w(t)dt \qquad \int_{t_3}^{t_5} c(t)dt$$

2. (10 points) The graph of the *derivative* of the continuous function $M(x)$ is given below.



Using the fact that $M(-4) = -2$, sketch the graph of $M(x)$ on the axes below. Give the coordinates of all critical points, inflection points and endpoints of M on the interval [-4,4].



3. The following questions are each meant to have short computation times. Each question is worth 4 points.

   (a) If $f(x)$ is even and $\int_{-2}^{2} (f(-x) - 3)\, dx = 8$, find $\int_{0}^{2} f(x)\, dx$.

   (b) The average value of the function $g(x) = 10/x^2$ on the interval $[c, 2]$ is equal to 5. Find the value of c.

   (c) If people are buying UMAir Flight 123 tickets at a rate of $R(t)$ tickets/hour (where t is measured in hours since noon on December 15, 2008), explain in words what $\int_{3}^{27} R(t)\, dt$ means in this context.

   (d) Suppose that the function $N = f(t)$ represents the total number of students who have turned in this exam t minutes after the beginning of the exam. Interpret $(f^{-1})'(325) = 2$.

   (e) Find k so that the function $h(x)$ below is continuous for all x.

$$h(x) = \begin{cases} x^2 + 1, & x \leq 1 \\ 6\sin(\pi(x - 0.5)) + k, & x > 1 \end{cases}$$

4. Consider the function $f(x) = x^3 \ln x$.

   (a) (4 points) Use the general expression for a left-hand sum using 4 subdivisions to write an approximation for

   $$\int_1^3 x^3 \ln x \, dx$$

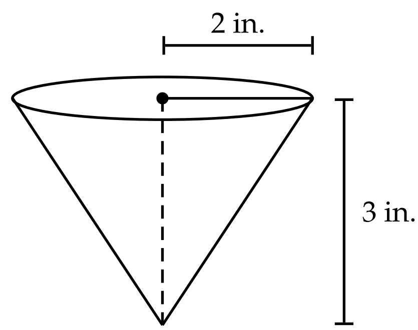
   —*i.e.*, express each term of the left-hand sum, using the given function. There is no need to evaluate the sum.

   (b) (3 points) Show that $\displaystyle\int x^3 \ln x \, dx = \frac{x^4}{4} \ln x - \frac{x^4}{16} + C$. Show your work.

   (c) (4 points) Use the Fundamental Theorem of Calculus and part (b) to find the exact value of $\displaystyle\int_1^3 x^3 \ln x \, dx$. Leave your answer in *exact* form—in other words, do not convert to a decimal. Again, show your work.

5. (15 points) Suppose that you are brewing coffee and that hot water is passing through a special, cone-shaped filter (see below). Assume that the height of the conic filter is 3 in. and that the radius of the base of the cone is 2 in. If the water is flowing out of the bottom of the filter at a rate of 1.5 in^3/min when the remaining water in the filter is 2 in. deep, how fast is the depth of the water changing at that instant?

   [Note: if d is depth of the water in the cone and the radius is r, the volume is given by $V = \frac{1}{3}\pi r^2 d$.]

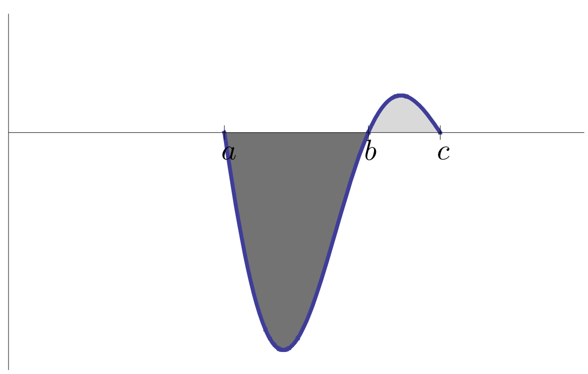


6. (7 points) The derivative of a continuous function g is given by

$$g'(x) = \frac{e^{-2x}(x+2)(x-3)^2}{(x-5)^{1/3}}.$$

Determine all critical points of g, and classify each as a local maximum, a local minimum, or neither. Carefully explain your reasoning for each classification.

7. (8 points) Use the following figure, which shows a graph of $f(x)$, to find each of the indicated integrals, given that the first area (with the darker shading) is 12 units and the second area is (with the lighter shading) is 3 units.



$(a) \int_a^b f(x)\,dx$ ____________ $(b) \int_a^c |f(x)|\,dx$ ____________

$(c) \int_c^a f(x)\,dx$ ____________ $(d) \int_a^a 2(f(x)+3)\,dx$____________

8. At the Michigan-Ohio State basketball game this year, the Michigan Band discovers that the amount of time it spends playing "Hail to the Victors" has a direct impact on the number of points our team scores. If the band plays for x minutes, then the Wolverines will score

$$W(x) = -.48x^2 + 7.2x + 63$$

points in the game. Assume that the band can play for a maximum of 10 minutes.

(a) (5 points) How long should the band play to maximize the number of points Michigan scores? Show your work and explain.

(b) (5 points) The band affects how many points Ohio State scores as well. When the U-M band plays for x minutes the Buckeyes score

$$B(x) = -x^2 + 8x + 84$$

points in the game. Find the number of minutes the band should play to maximize the margin of victory for Michigan (*i.e.,* the points by which Michigan wins or loses). Again, please show all work.

(c) (2 points) What will be the score of the game for the case you found in part (b)?

Michigan:	____________	points
Ohio State:	____________	points