Math 115 — First Midterm

October 13, 2009

Name: ______________________________

Instructor: ______________________ Section: ____________

1. **Do not open this exam until you are told to do so.**
2. This exam has 9 pages including this cover. There are 9 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Include units in your answer where that is appropriate.
6. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
8. **Turn off all cell phones and pagers**, and remove all headphones.

Problem	Points	Score
1	12	
2	12	
3	12	
4	15	
5	16	
6	10	
7	6	
8	9	
9	8	
Total	100	

1. [12 points] For each problem below, circle all of the statements that MUST be true. (The three parts (a)–(c) are independent of each other. No explanations are necessary.)

a. [5 points] Suppose f is an increasing differentiable function with domain $(-\infty, \infty)$ so that $f(1) = 1$ and $f(-1) = -1$.

- ∘ f is linear.
- ∘ There is a number c so that $f(c) = 0$.
- ∘ $\lim_{x\to 1} f(x) = 1$
- ∘ $\lim_{x\to\infty} f(x) = \infty$
- ∘ $f'(1) \geq 0$

b. [3 points] Suppose $g(t)$ is the mass (in grams) of mold on a wedge of cheese in a refrigerator t days after it was abandoned. This mass grows exponentially as a function of time for two weeks, when it is finally thrown away.

- ∘ The graph of g is concave up.
- ∘ The continuous growth rate of g is less than the daily growth rate.
- ∘ The amount of time it takes for the mass of mold on the cheese to triple is 1.5 times the amount of time it takes for it to double.

c. [4 points] If $f(x) = \dfrac{g(x)}{h(x)}$ and $h(3) = 0$ then

- ∘ The graph of f has a vertical asymptote at $x = 3$.
- ∘ 3 is not in the domain of f.
- ∘ f is not continuous on $[-2, 2]$.
- ∘ $\lim_{x\to 3} f(x)$ does not exist.

2. [12 points] Suppose that the line tangent to the graph of $f(x)$ at $x = 3$ passes through the points $(1, 2)$ and $(5, -4)$.

a. [3 points] Find $f'(3)$.

b. [3 points] Find $f(3)$.

c. [3 points] Estimate the value of $f(2.9)$.

d. [3 points] If the graph of f is concave up, is your estimate in part (c) an overestimate, an underestimate, or can you not tell? Explain or demonstrate your answer graphically.

3. [12 points] The popularity of baby names varies over time; the names that are popular one year may not be popular at all within a few years. The popularity of baby names beginning with the letter I appears to be periodic. In 1885, approximately $16,000$ per million babies born had first names beginning with the letter I. Their popularity began decreasing at that time and decreased until 1945, when the number had dropped to a low of $2,200$ per million. In 2005 it was back to $16,000$ per million babies born.

Let $B(t)$ denote the popularity of names beginning with the letter I, in thousands of babies per million babies born, t years after 1885. Assume that $B(t)$ is a sinusoidal function.

a. [6 points] Sketch the graph of $B(t)$. (Remember to clearly label your graph.)

b. [6 points] Find a formula for $B(t)$.

4. [15 points] The H1N1 flu virus arrived in the US last spring. Data from the CDC for the region including Michigan, Minnesota, Illinois, Indiana, Wisconsin, and Ohio is shown in the table below. $H(t)$ denotes the cumulative number of cases of H1N1 flu in this region t weeks after August 15, 2009.

t	0	1	2	3	4	5	6
$H(t)$	8266	8314	8365	8482	8632	8903	9165

a. [2 points] Evaluate and interpret $H(5)$.

b. [2 points] Why might it be reasonable to assume that H is invertible for $0 \le t \le 6$?

c. [3 points] Assuming H is invertible, give the practical meaning of $H^{-1}(8500)$.

d. [3 points] Estimate $H'(5)$.

e. [5 points] Assuming H is invertible, estimate and give the practical meaning of $(H^{-1})'(8500)$.

5. [16 points] Since it was first introduced, the number of users of the internet worldwide has increased dramatically. Let $I(t)$ denote the number (in millions) of worldwide internet users t years after 1995. Then $I(t)$ is given by the formula

$$I(t) = \begin{cases} 16(361/16)^{t/5} & \text{if } 0 \le t \le 5 \\ 361(1.18)^{t-5} & \text{if } 5 < t \le 10 \\ A + 10(t-10) & \text{if } t > 10 \end{cases}$$

a. [3 points] Find A so that $I(t)$ is continuous.

b. [4 points] Find the continuous growth rate of $I(t)$ in the year 1997.

c. [3 points] Find the average rate of change of the number of internet users between 1995 and 2000.

d. [6 points] Use the definition of the derivative to numerically estimate (i) $I'(7)$ and (ii) $I'(10)$.

6. [10 points] The force F, in Newtons, between two atoms a distance r femtometers (fm) apart in a molecule is given by $F(r) = b\left(\frac{a^2}{r^3} - \frac{a}{r^2}\right)$ for some positive constants a and b.

Note: Your answers below might involve the constants a and b.

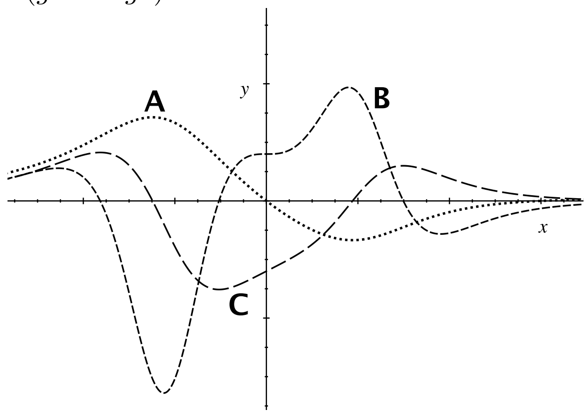
a. [3 points] Find and interpret any horizontal intercept(s) of the graph of $F(r)$.

b. [3 points] Find any asymptote(s) of the graph of $F(r)$.

c. [4 points] Give the practical interpretation of $F'(1) = -1.2 \times 10^{-9}$.

7. [6 points] Consider the function $W(t) = 3\ln\left(\sin(t)^2 + 2\right)$. Write down the limit definition of $W'(\pi)$. (You do not need to estimate or compute the derivative.)

8. [9 points] The three graphs labeled A, B, and C below depict a function g along with its first and second derivatives (g' and g''). Determine which is which.



Your answer to parts (a)–(c) should be a single legible capital letter (A, B, or C).

a. [2 points] The graph of g is labeled _______.

b. [2 points] The graph of g' is labeled _______.

c. [2 points] The graph of g'' is labeled _______.

d. [3 points] Briefly explain your reasoning.

9. [8 points] On the axes provided below, sketch the graph of a single function f satisfying all of the following:

- $f''(x) > 0$ for $x < -2$.
- The graph of f has a vertical asymptote at $x = -2$.
- $f'(-1) = -3$
- $\lim_{x \to 0} f(x) = 2$
- $f(0) = -2$
- f is continuous but not differentiable at $x = 1$.
- $f'(x) > 0$ for $x > 3$.
- $\lim_{x \to \infty} f(x) = 4$

Remember to clearly label your graph.