Math 115 — Final Exam

December 17, 2009

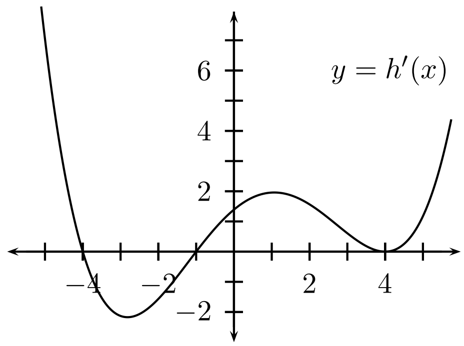
Name: ____

Instructor: ______________________________ Section: ______________

1. **Do not open this exam until you are told to do so.**
2. This exam has 9 pages including this cover. There are 8 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Include units in your answer where that is appropriate.
6. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
8. **Turn off all cell phones and pagers**, and remove all headphones.
9. **Use the techniques of calculus to solve the problems on this exam.**

Problem	Points	Score
1	13	
2	14	
3	12	
4	12	
5	13	
6	12	
7	12	
8	12	
Total	100	

1. [13 points] The graph of the **derivative**, $h'(x)$, of a continuous function h is shown below:



a. [3 points] Approximate the x-coordinates of all critical points of h in the interval $(-5, 5)$, and classify each as either a local maximum, a local minimum, or neither.

b. [3 points] Approximate the x-coordinate(s) of any inflection point(s) of h in the interval $(-5, 5)$.

c. [2 points] Approximate the value(s) of x on the interval $[-5, 5]$ where h attains its global maximum.

d. [2 points] Approximate the value(s) of x on the interval $[-5, 5]$ where h attains its global minimum.

e. [3 points] If $h(1) = 3$, find the best linear approximation to $h(x)$ at the point $x = 1$. Is this linear approximation an underestimate or an overestimate of h for points near $x = 1$? Explain.

2. [14 points] Let $C(t)$ be the temperature, in degrees Fahrenheit, of a warm can of soda t minutes after it was put in a refrigerator. Suppose $C(10) = 62$.

a. [3 points] Assuming C is invertible, give a practical interpretation of the statement $C^{-1}(45) = 40$.

b. [3 points] Give a practical interpretation of the statement $C'(10) = -0.4$.

c. [3 points] Give a practical interpretation of the statement $\int_0^{10} C'(t)\,dt = -5$.

d. [2 points] Assuming the statements in parts (a)-(c) are true, determine $C(0)$.

e. [3 points] What is the practical meaning of $\int_0^1 C(t)\,dt$?

3. [12 points]
Use the information in the table below to answer (a) - (c):

x	-3	-2	-1	0	1	2	3
$f(x)$	1	3	2	-1	-3	-2	0
$f'(x)$	3	2	-2	-3	-1	1	2
$g(x)$	2	3	1	2	4	3	1
$g'(x)$	1	-2	2	3	-1	0	-3

a. [4 points] If $h(x) = g\left(f(e^{\pi} \ln x)\right)$, find $h'(1)$. (Give an exact answer.)

b. [4 points] If $j(x) = \sin^2\left(\frac{3f(x)}{2}\right)$, find $j'(-2)$. (Give an exact answer.)

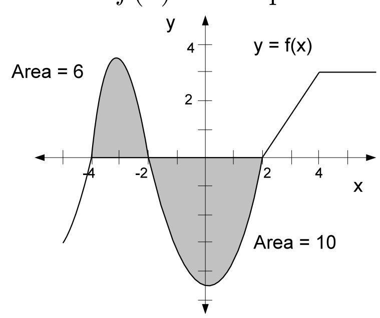
c. [4 points] Give an exact answer for $\displaystyle\int_{-3}^{2} \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}\, dx$, assuming $g(x) \neq 0$.

4. [12 points]

a. [5 points] If the average value of a continuous function g on $[1,8]$ is 3, find

$$\int_{-1}^{6} 3\,(g(x+2)) + 5\,dx.$$

Use the following graph of a function $f(x)$ to compute the quantities in parts (b)–(d) below.

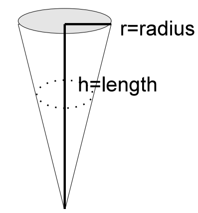


b. [2 points] $\displaystyle\int_{-4}^{2} f(x)\,dx$

c. [3 points] The area between the graph of $f(x)$ and the x–axis for $-4 \le x \le 5$ if the units on the axes are centimeters.

d. [2 points] $\displaystyle\int_{3}^{5} f'(x)\,dx$

5. [13 points] A cone-shaped icicle is dripping from above the entrance to Dennison Hall. The icicle is melting at a rate of 1.2 cm^3 per hour. At 10:00 a.m., the icicle was 25 cm long and had a 2 cm radius at its widest point. Assume that the icicle keeps the same proportions as it melts. [Note: the volume of a cone is $V = \frac{1}{3}\pi r^2 h$.]



a. [5 points] Determine the rate at which the length of the icicle is changing at 10:00 a.m.

b. [4 points] At what rate is the radius of the icicle changing at 10:00 a.m.?

c. [4 points] Let $V(t)$ and $r(t)$ denote the volume and radius, respectively, of the icicle t hours after 10:00 a.m. Assume that the icicle continued to melt from $t = 0$ (10:00 a.m.) to $t = M$. Circle all of the statements below that must be true if "After the icicle began dripping at 10:00 a.m., it took exactly M hours for the icicle to melt completely." [Circle the entire expression, and be certain that your circled answers are VERY clear!!]

i. $\displaystyle\int_0^M V'(t)\,dt > \int_0^{M/2} V'(t)\,dt$

ii. $\displaystyle\int_0^M V'(t)\,dt = 0$

iii. $\displaystyle\int_0^M V'(t)\,dt = -V(0)$

iv. $\displaystyle\int_0^2 r(t)\,dt = 0$

v. $\displaystyle\int_0^M r(t)\,dt = -2$

vi. $\displaystyle\int_0^M r'(t)\,dt = -2$

vii. $\displaystyle\int_2^0 V'(r)\,dr = M$

viii. $\displaystyle\int_0^M h(t)\,dt = 0$

6. [12 points] The rate $q(t)$ at which cars passed through the intersection of Main Street and Huron after a football game is presented in the table below.

t (in minutes after the game ended)	0	20	40	60	80	100	120
$q(t)$ (in cars per minute)	10	15	19	21	20	17	13

a. [4 points] What is the meaning of $\int_0^{120} q(t)\,dt$? Using a left Riemann sum and $n = 6$, estimate $\int_0^{120} q(t)\,dt$. (Write out the terms of your sum.)

b. [2 points] Write an expression for the average rate at which cars passed through the intersection for the first two hours after the game ended.

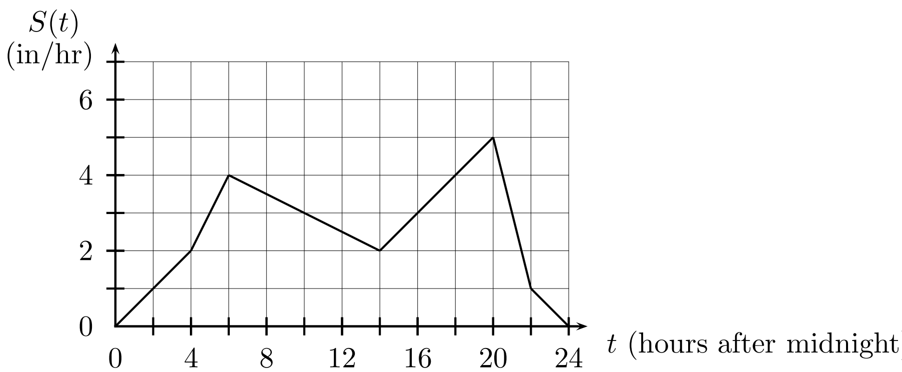
c. [3 points] Estimate $q'(30)$.

d. [3 points] If $Q(t)$ denotes the total number of cars that have passed through the intersection t minutes after the game ended, find and interpret $Q'(60)$.

7. [12 points] After an unusual winter storm, the EPA is concerned about potential contamination of a river. A new researcher has been assigned the task of taking a sample to test the water quality. She tried to get as close to the river as possible in her car, but was forced to park a feet away. She also cannot get closer to the lab by car. She needs to walk to the river, retrieve a water sample, and then walk the sample to a lab located $4a$ feet down the river and $2a$ feet from the river bank.

If the researcher wants to walk as short a distance as possible, what path should she take as she walks from her car to the river and then from the river to the lab?

8. [12 points]
The graph below gives the rate $S(t)$, in inches per hour, of snow fall t hours after midnight along a major thoroughfare in Ann Arbor. Beginning at 8:00 a.m., the city truck began removing snow at the rate of 2 in/hr. [Salting had been halted, as a consequence of economic conditions in Michigan.] Assume that there was no snow on the road prior to midnight.



a. [2 points] How deep was the snow at 2:00 p.m.?

b. [2 points] At what time was the snow falling the fastest?

c. [2 points] At what time was the snow deepest?

d. [2 points] At what time was the depth of the snow on the ground increasing fastest?

e. [2 points] What is the average rate at which snow fell between 4 am and 2 pm?

f. [2 points] Write an expression for the average depth of the snow on the ground between 5 am and 8 am.