Math 115 — Second Midterm

November 16, 2010

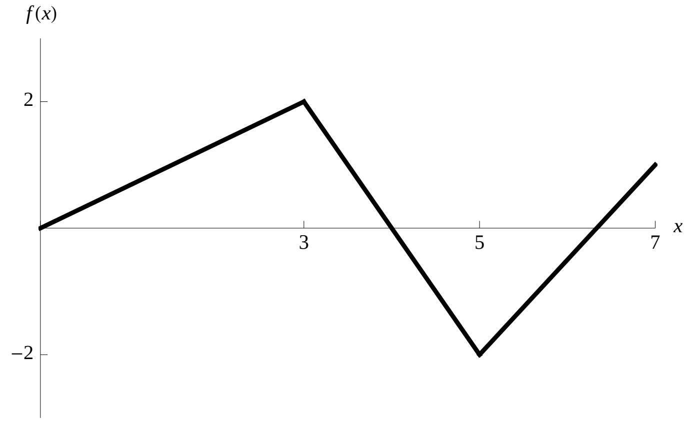
Name: ______________________________

Instructor: ______________________________ Section: ______________

1. **Do not open this exam until you are told to do so.**
2. This exam has 9 pages including this cover. There are 8 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Include units in your answer where that is appropriate.
6. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
8. **Turn off all cell phones and pagers**, and remove all headphones.

Problem	Points	Score
1	10	
2	14	
3	15	
4	14	
5	12	
6	10	
7	15	
8	10	
Total	100	

1. [10 points] Given below is a graph of a function $f(x)$ and a table for a function $g(x)$.



x	0	1	2	3	4
$g(x)$	4	3	1	2	$\frac{20}{3}$
$g'(x)$	-2	$-\frac{5}{2}$	$\frac{1}{2}$	3	$-\frac{1}{3}$

Give answers for the following or write "Does not exist." No partial credit will be given.

i) $\dfrac{d}{dx}f(g(x))$ at $x = 0$

ii) $\dfrac{d}{dx}[f(x)g(x)]$ at $x = 2$

iii) $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right]$ at $x = 4$

iv) $\dfrac{d}{dx}[g(f(x))]$ at $x = 3$

v) $f(g'(3))$

2. [14 points] The table for the *derivative* of a function h with continuous first derivative is given below. Assume that between each consecutive value of x, the derivative h' is either increasing or decreasing. For each statement below, indicate whether the statement is true, false, or cannot be determined from the information given. No partial credit will be given.

x	-4	-3	-2	-1	0	1	2	3	4
$h'(x)$	2	3	1	-3	-4	-2	0	2	1

a.) The function h has a local maximum on the interval $-2 < x < -1$.

True False Not enough information

b.) The function h is negative on the interval $-1 < x < 1$.

True False Not enough information

c.) The function h is concave up on the interval $0 < x < 4$.

True False Not enough information

d.) The function h is decreasing on the interval $-3 < x < -2$.

True False Not enough information

e.) The function h has an inflection point on the interval $-1 < x < 1$.

True False Not enough information

f.) The derivative function, h', has a critical point at $x = 2$.

True False Not enough information

g.) The second derivative function, h'', is positive on the interval $0 < x < 3$.

True False Not enough information

3. [15 points] Answer "True" or "False" for each of the following, and **include a brief explanation of your answer.** A picture may be sufficient for an explanation, if appropriate. The functions h, h', m and m' referred to in the problem are all differentiable on their domain. The letters a and b represent constants. (Note: Answer "True" only if the statement is *always* true.)

i) If $y = h'(x)m(x) - h(x)m'(x)$, then $\dfrac{dy}{dx} = h''(x)m(x) - h(x)m''(x)$.

True False

ii) If $m''(a) = 0$, then $m(x)$ has an inflection point at $x = a$.

True False

iii) If $h''(x) > 0$ on the interval $[a, b]$ and $h(a) > h(b)$, then $h(a)$ is the absolute maximum value of $h(x)$ on $[a, b]$.

True False

iv) There exists a continuous function $f(x)$ which is not differentiable at $x = 0$ with a local maximum at $(0, 5)$.

True False

v) The function $g(x) = e^{-(x-a)^2/b}$ has a local maximum at $x = b$.

True False

4. [14 points] It is projected that the number of marine plant and animal species on earth will decrease by 40% by the year 2050. The current (2010) instantaneous rate of marine species loss is 80,000 species per year.

a. [6 points] Assuming the number of marine species is modeled by an exponential function, write an exponential function $M = f(t)$ which outputs the total number of marine species t years after 2010.

b. [3 points] According to your model, about how many marine species are there currently (in 2010)? About how many will there be in 2050?

c. [5 points] In what year will there be half as many marine species as there are currently (in 2010)? Using derivatives, approximate how many marine species will be lost that year.

5. [12 points] Suppose a curve in the plane is given by the equation

$$\sin(\pi xy) = y - 1.$$

a. [3 points] Verify that the point $(x, y) = (1, 1)$ is on the curve.

b. [5 points] Calculate $\frac{dy}{dx}$.

c. [4 points] Find the equation for the tangent line to the curve at the point $(1, 1)$.

6. [10 points] Calvin is stuck in the desert, and he needs to build a cube out of cactus skins to hold various supplies. He wants his cube to have a volume of 8.1 cubic feet, but he needs to figure out the side length to cut the cactus skins the right size. He has forgotten his trusty calculator, so he decides to figure out the side length of his cube using calculus.

a. [5 points] Find a local linearization of the function $f(x) = (x + 8)^{1/3}$ at $x = 0$.

b. [3 points] Use your linearization to approximate $(8.1)^{1/3}$.

c. [2 points] Should your approximation from part **b.** be an over-estimate or an under-estimate? Why?

7. [15 points] Suppose a is a positive constant and

$$f(x) = 2x^3 - 3ax^2.$$

a. [10 points] Find the absolute maximum and minimum values of $f(x)$ on the closed interval $[-a, \frac{3}{2}a]$. Specify all x values where the maximum and minimum value are achieved.

b. [5 points] Find all inflection points of $f(x)$.

8. [10 points] Farmer Fred is designing a fence next to his barn for his grass-fed herd of cattle. The fence will be rectangular in shape with wooden fence on three sides and a chain link fence on the side closest to his barn. The wooden fence costs $6 per foot and the chain link fence costs $3 per foot. If he wants the fenced area to be 40,000 square feet, what should the dimensions of his fence be in order to minimize his total cost?