Math 115 — First Midterm

October 11, 2011

Name: _____

Instructor: _____   Section: _____

1. **Do not open this exam until you are told to do so.**

2. This exam has 10 pages including this cover. There are 10 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.

3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.

4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.

5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it. Include units in your answer where that is appropriate.

6. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.

7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.

8. **Turn off all cell phones and pagers**, and remove all headphones.

9. You must use the methods learned in this course to solve all problems.

Problem	Points	Score
1	12	
2	12	
3	12	
4	12	
5	6	
6	5	
7	10	
8	9	
9	12	
10	10	
Total	100	

1. [12 points] For each part below, give an explicit formula for a function which satisfies the given properties, if one exists. If such a function does not exist, explain why. Be sure to clearly indicate your final answer for each part.

   a. [3 points] A continuous function, f, which is not differentiable.

   b. [3 points] A cubic polynomial, p, with two x-intercepts.

   c. [3 points] A continuous function, c, satisfying $\lim_{x \to 0^+} c(x) = -1$ and $\lim_{x \to 0^-} c(x) = 1$.

   d. [3 points] A rational function, r, with a vertical asymptote at $x = 1$ and a horizontal asymptote at $y = 1$.

2. [12 points] The Facebook Data team has decided to track the University of Michigan network status updates that mention football in order to see which days to show ads for tailgating supplies. Starting at 1pm Saturday, they measure an aggregate Football Status Factor by calculating the percentage of status updates which mention any of a number of designated football terms every hour. They notice very quickly that the data is sinusoidal with period 168 hours (the number of hours in a week). Suppose $F(t)$ is this percentage, t hours after 1pm Saturday.

    a. [2 points] If the maximum percentage is 96% at 1pm Saturday, and the minimum is 28% attained 84 hours later, compute the following quantities:

        1. Midline

        2. Amplitude

    b. [6 points] Using the values computed above, find a formula for $F(t)$.

    c. [4 points] Suppose advertisers want to advertise when the rate at which people are talking about football is increasing the fastest. What time range would you recommend to them and why? Use a graph of $F(t)$ to justify your answer.
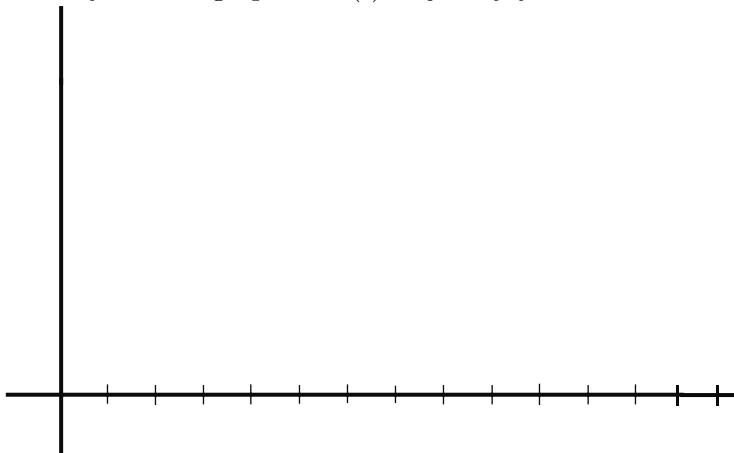
3. [12 points] A zombie plague has broken out in Ann Arbor. As a nurse in the University of
Michigan hospital, you saw the person with the first case of the plague, patient zero.

    a. [2 points] In order to keep track of the growing zombie population in Ann Arbor, you
collected the following data:

Days after patient zero	0	6	9	12
Number of Zombies	1	9	27	81

Would a linear function or an exponential function be the best model? Why?

    b. [4 points] Write a function $Z(t)$ of the appropriate type to model the growth of the zombie
population with t measured in days after patient zero.

    c. [3 points] The population of North America is approximately 530,000,000 people. Using
your model, how long will it take until all but one person are infected?

    d. [3 points] Using your table, approximate the instantaneous rate of change of the zombie
population on the ninth day.

4. [12 points] The Twitter Celebrity Index (TCI) measures the celebrity of Twitter users; the function $T(x)$ takes the number of followers (in millions) of a given user and returns a TCI value from 0 to 10. Below is a graph of this function.
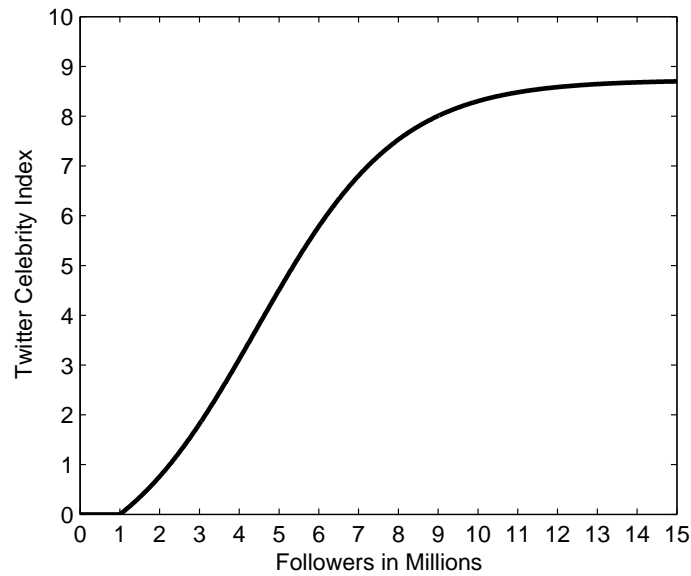
Use the graph above to help you answer the following questions.

a. [3 points] Explain in practical terms what $T(13.72) = 8.67$ means.

b. [3 points] Explain in practical terms what $T^{-1}(4.25) = 4.88$ means.

c. [3 points] Explain in practical terms what $T'(10) = 0.2278$ means.

d. [3 points] Explain in practical terms what $(T^{-1})'(7.238) = 0.71$ means.

5. [6 points] Find a number k so that the following function is continuous on any interval.

$$j(t) = \begin{cases} (t+4)^3 & t < -2 \\ kt & t \geq -2 \end{cases}$$

Using your value of k, explain why this function is continuous on any interval.

6. [5 points]   Using the limit definition of the derivative, write an explicit expression for the derivative of the function $E(x) = x^{\cos x}$ at $x = 2$. Do not try to calculate this derivative.

7. [10 points] On the axes below sketch a well-labeled graph of a continuous function, g, which satisfies all of the following properties.

   a. $g'(x) = 2$ for $1 < x < 2$

   b. $g'(x) = -2$ for $2 < x < 3$

   c. $g(0) = -1$

   d. $g(1) = 0$

   e. g is decreasing for $x > 3$

   f. $g''(x) < 0$ for $x > 3$

   g. g is concave down for $x < 1$

x
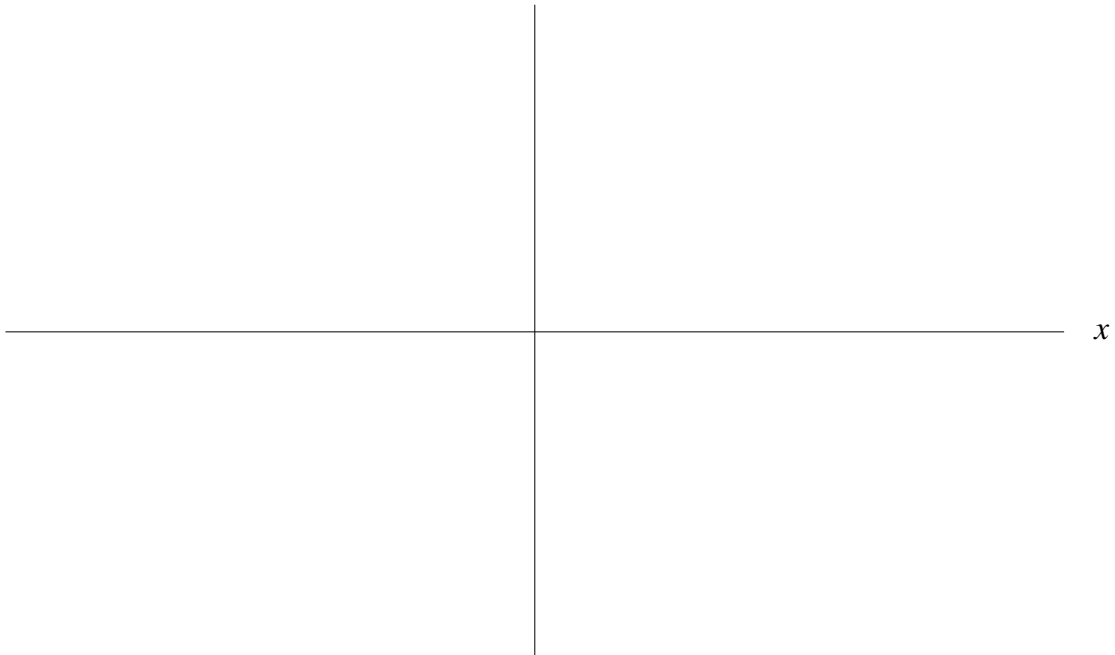
8. [9 points]   The graph below shows a runner's distance, p, in miles from her starting point t minutes after she began to run.
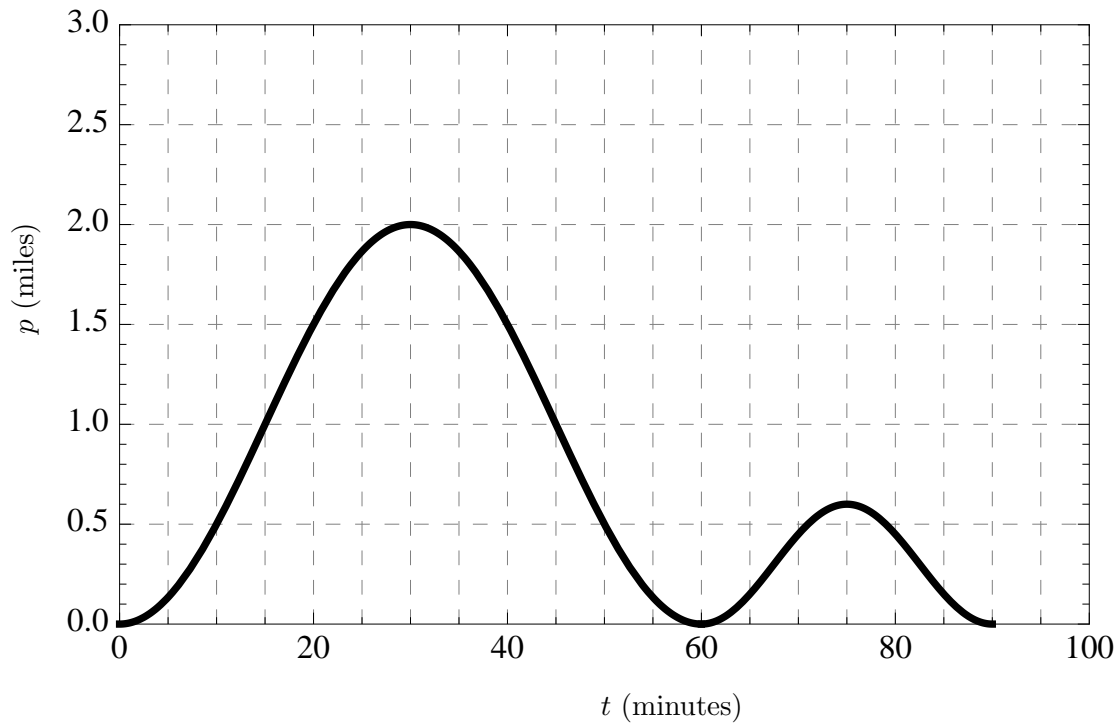
Using the graph, estimate the following.

a. [3 points]   All times during her run where her velocity was zero.

$t =$ _____

b. [2 points]   Her average velocity over the first 45 minutes of her run.

velocity= _____

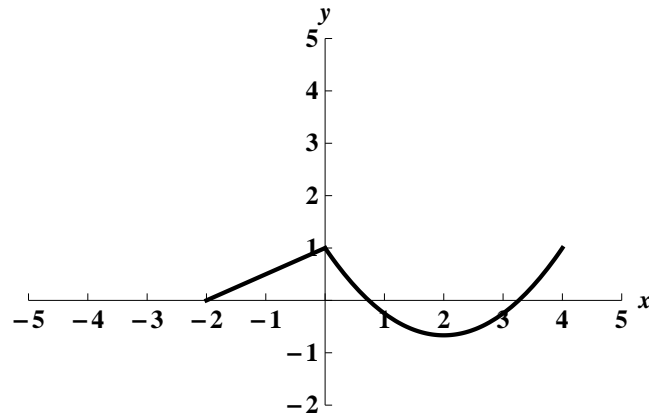**c.** [2 points]   Her average speed over the first 45 minutes of her run.

speed= _____

d. [2 points]   Her velocity 80 minutes after she began running.
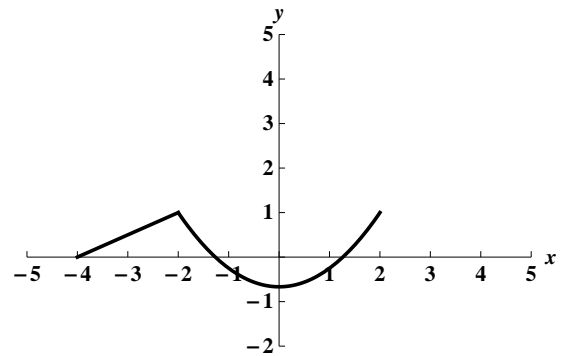
velocity= _____

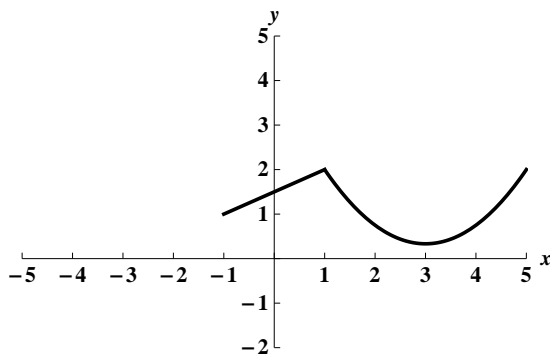**9.** [12 points] Consider the graph below of $g(x)$:

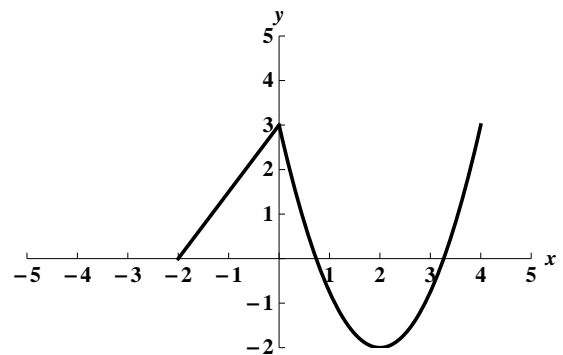The four graphs below are shifts or stretches of $g(x)$. Write each function below in terms of $g(x)$.
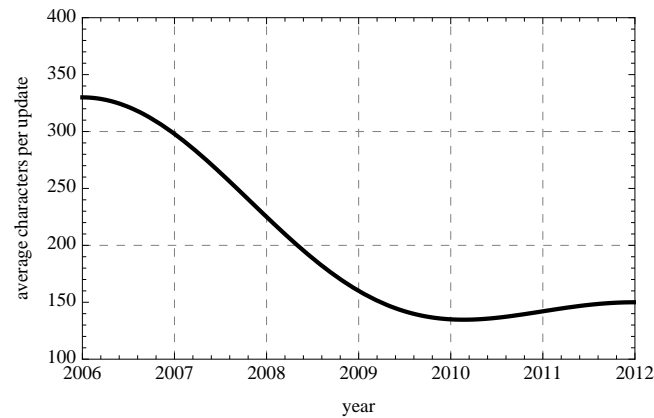

$h(x) = $ _____

$k(x) = $ _____
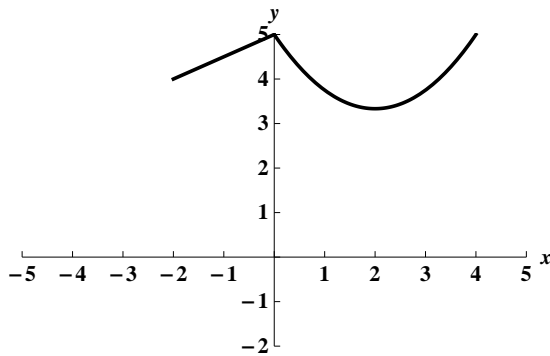
$\ell(x) = $ _____

$m(x) = $ _____

10. [10 points] Facebook tracks the average number of characters used by its users to write their status updates. Below is the graph for a random (talkative) user from the beginning of 2006 to the beginning of 2012. Use the graph to answer the following questions.

a. [2 points] When were this user's status updates the longest? How long were they?

b. [3 points] When was the length of the user's status updates decreasing? Increasing?

c. [2 points] When was the length of status updates shrinking the fastest?

d. [3 points] Is this function continuous? Is it invertible? Justify your answer.