On my honor, as a student,
I have neither given nor received
unauthorized aid on this academic work. **Initials:** ______

Do not write in this area

Math 115 — Second Midterm — November 12, 2018

Your Initials Only: ______ Your U-M ID # (not uniqname): ______________

Instructor Name: ______________ Section #: ______

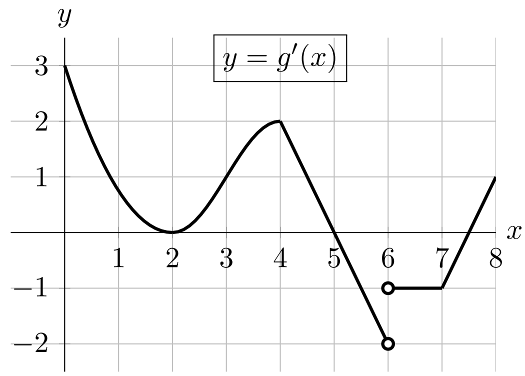
1. **Do not open this exam until you are told to do so.**
2. **Do not write your name anywhere on this exam.**
3. This exam has 11 pages including this cover. There are 11 problems.
   Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
4. Do not separate the pages of this exam. If they do become separated, write your UMID (not name) on every page and point this out to your instructor when you hand in the exam.
5. Note that the back of every page of the exam is blank, and, if needed, you may use this space for scratchwork. Clearly identify any of this work that you would like to have graded.
6. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
7. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it.
8. The use of any networked device while working on this exam is not permitted.
9. You may use any one calculator that does not have an internet or data connection except a TI-92 (or other calculator with a "qwerty" keypad). However, you must show work for any calculation which we have learned how to do in this course.
   You are also allowed two sides of a single $3'' \times 5''$ notecard.
10. For any graph or table that you use to find an answer, be sure to sketch the graph or write out the entries of the table. In either case, include an explanation of how you used the graph or table to find the answer.
11. Include units in your answer where that is appropriate.
12. Problems may ask for answers in *exact form*. Recall that $x = \sqrt{2}$ is a solution in exact form to the equation $x^2 = 2$, but $x = 1.41421356237$ is not.
13. **Turn off all cell phones, smartphones, and other electronic devices**, and remove all headphones, earbuds, and smartwatches. Put all of these items away.
14. You must use the methods learned in this course to solve all problems.

Problem	Points	Score
1	17	
2	6	
3	12	
4	12	
5	9	
6	4	

Problem	Points	Score
7	5	
8	7	
9	13	
10	7	
11	8	
Total	100	

1. [17 points] Let $g(x)$ and $h(x)$ be two functions. The graphs of $g'(x)$ and $h''(x)$ are shown below.

At right is the graph of $y = g'(x)$, the **derivative** of $g(x)$.
Assume that $g(x)$ is a **continuous** function.
Use the graph to answer the questions below. Circle <u>all</u> correct answers.



a. [2 points] At which of the following values of x is $g(x)$ <u>not</u> differentiable?

$x = 2$ $x = 4$ $x = 5$ $x = 6$ $x = 7$ NONE OF THESE

b. [2 points] For which of the following values of x does $g(x)$ have a local maximum?

$x = 2$ $x = 4$ $x = 5$ $x = 6$ $x = 7.5$ NONE OF THESE

c. [2 points] For which of the following values of x does $g(x)$ have an inflection point?

$x = 2$ $x = 3$ $x = 4$ $x = 5$ $x = 7.5$ NONE OF THESE

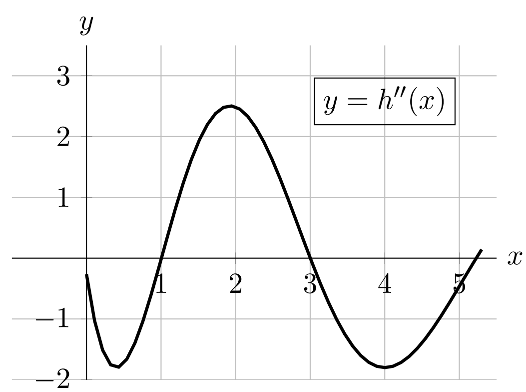
d. [2 points] On which of the following intervals is $g(x)$ linear?

$(0, 2)$ $(4, 6)$ $(6, 7)$ $(6, 8)$ $(7, 8)$ NONE OF THESE

e. [2 points] For which of the following values of x does $g(x)$ attain a global maximum on the interval $[1, 7]$?

$x = 2$ $x = 4$ $x = 5$ $x = 6$ $x = 7$ NONE OF THESE

Use the graph of $y = h''(x)$, the **second derivative** of $h(x)$, to answer the questions below. Circle <u>all</u> correct answers.



f. [2 points] Over which of the following intervals is $h(x)$ concave up on the entire interval?

$(0, 1)$ $(1, 3)$ $(2, 4)$ $(4, 5)$ NONE OF THESE

g. [2 points] On which of the following intervals is the function $h'(x)$ (the derivative of $h(x)$) decreasing over the entire interval?

$(0, 1)$ $(1, 3)$ $(2, 3)$ $(4, 5)$ NONE OF THESE

h. [3 points] If $h'(4) = 0$, which of the following statements <u>must</u> be true?

A. $x = 4$ is a local maximum of $h(x)$.

B. $x = 4$ is a local minimum of $h(x)$.

C. $x = 4$ is an inflection point of $h'(x)$.

D. $x = 4$ is a critical point of $h(x)$.

E. $x = 4$ is an inflection point of $h(x)$.

F. NONE OF THESE

2. [6 points] Let A and B be constants and

$$k(x) = \begin{cases} 3x + \dfrac{B}{x} & \text{for} \quad 0 < x < 1 \\ Bx^2 + Ax^3 & \text{for} \quad 1 \leq x. \end{cases}$$

Find the values of A and B that make the function $k(x)$ differentiable on $(0, \infty)$. Show all your work to justify your answers. If there are no such values of A and B, write NONE.

Answer: $A =$ ________________ $B =$ ________________

3. [12 points] Assume the function $h(t)$ is invertible and $h'(t)$ is differentiable. Some of the values of the function $y = h(t)$ and its derivatives are shown in the table below

t	0	1	2	3	4
$h(t)$	-2	2	3	4	8
$h'(t)$	3.5	0.5	2.5	1.5	5
$h''(t)$	6	0.25	0.3	-0.4	0.6

Use the values in the table to compute the exact value of the following mathematical expressions. If there is not enough information provided to find the value, write NI. If the value does not exist, write DNE. **Show all your work.**

a. [3 points] Let $a(t) = h(t^2 - 5)$. Find $a'(3)$.

Answer: ________________

b. [3 points] Let $b(t) = \dfrac{h(t)}{t^2}$. Find $b'(4)$.

Answer: ________________

c. [3 points] Let $c(y) = h^{-1}(y)$. Find $c'(2)$.

Answer: ________________

d. [3 points] Let $g(t) = \ln(1 + 2h'(t))$. Find $g'(0)$.

Answer: ________________

4. [12 points] Suppose $r(x)$ is a differentiable function defined for all real numbers x. The derivative and second derivative of $r(x)$ are given by

$$r'(x) = (x+1)^3(x-2)^{4/5} \qquad \text{and} \qquad r''(x) = \frac{(x+1)^2(19x-26)}{5(x-2)^{1/5}}$$

a. [6 points] Find the x-coordinates of all critical points of $r(x)$ and all values of x at which $r(x)$ has a local extremum. Use calculus to find and justify your answers, and be sure to show enough evidence to demonstrate that you have found all local extrema. For each answer blank below, write NONE if appropriate.

Answer: Critcal points at $x =$ __________________

Local max(es) at $x =$ __________________ Local min(s) at $x =$ __________________

b. [6 points] Find the x-coordinates of all inflection points of $r(x)$. If there are none, write NONE. Use calculus to find and justify your answers, and be sure to show enough evidence to demonstrate that you have found all inflection points.

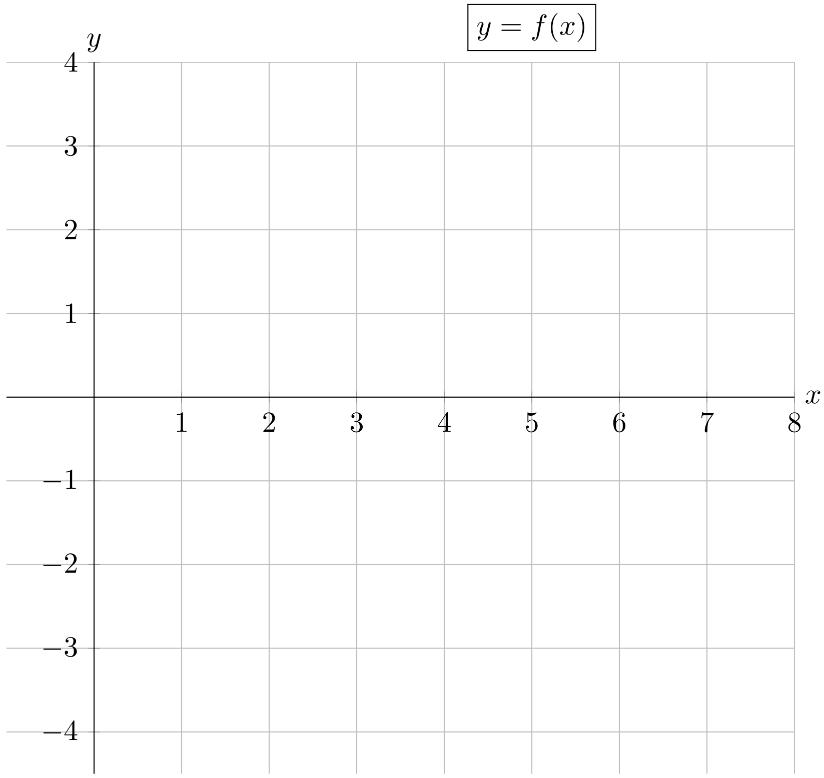
Answer: Inflection points at $x =$ __________________

5. [9 points] In each of the following questions, draw a graph satisfying **all** the properties listed. There may be many correct answers. Make sure that your graph clearly shows all of the properties listed.

a. [5 points] The function $f(x)$ satisfies each of the following properties:

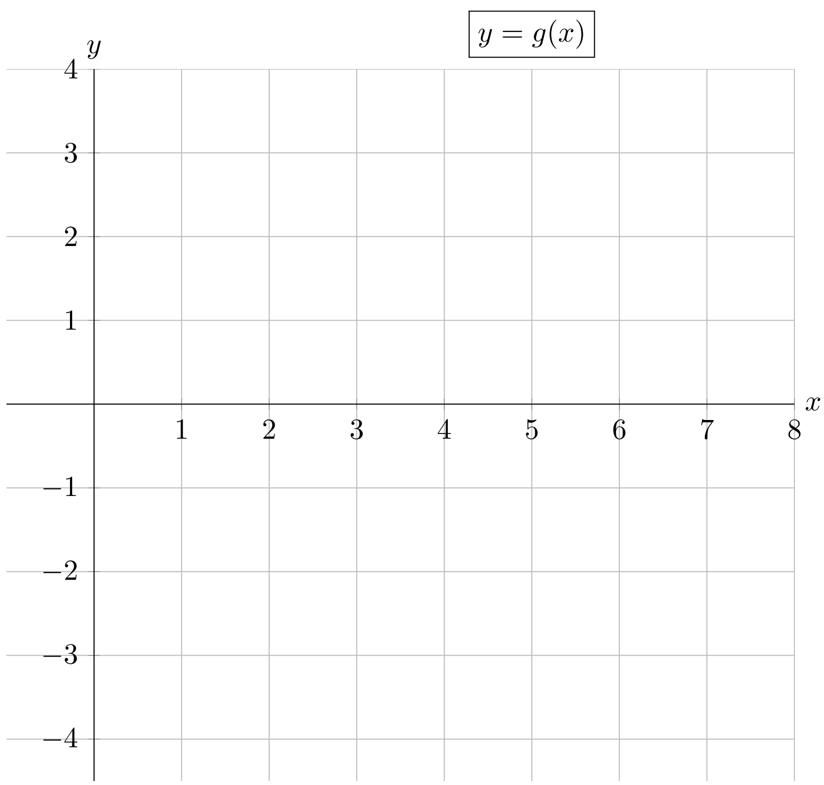
- $f(x)$ is continuous on $(0, 8)$.
- $f(x)$ has a local maximum at $x = 5$.
- $f''(x) < 0$ on $(4, 7)$.
- $\lim_{x \to 2^-} f'(x) = \infty$ and

  $\lim_{x \to 2^+} f'(x) = 0$

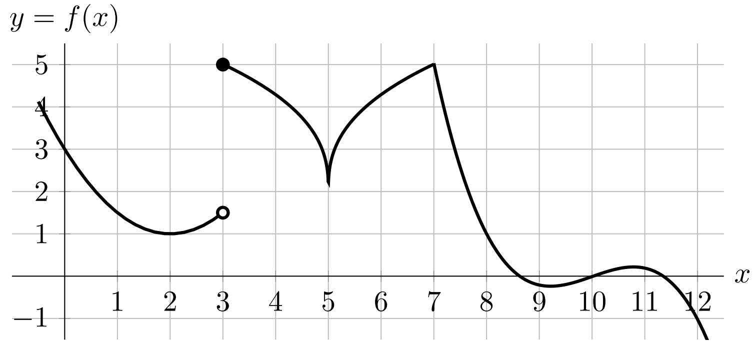


b. [4 points] The function $g(x)$ satisfies each of the following properties:

- $g(x)$ is defined on $(0, 8)$.
- $g(x)$ has an inflection point at $x = 3$.
- $g(x)$ is discontinuous at $x = 6$.
- $g(x)$ has a local maximum at $x = 6$.
- $g(x)$ has global maxima only at $x = 1$ and $x = 5$.



6. [4 points] The graph of the function $f(x)$ is shown below. Note that $f(x)$ has a vertical tangent line at $x = 5$.

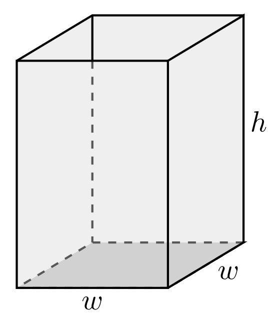


a. [2 points] On which of the following intervals does the function $f(x)$ satisfy the hypotheses of the Mean Value Theorem? Circle the correct answer(s).

[0,2] [1,3] [2,4] [3,5] NONE OF THESE

b. [2 points] On the interval $[8, 12]$ the hypotheses of the Mean Value Theorem are true for the function $f(x)$. What does the conclusion of this theorem say in this interval?
Answer:

7. [5 points]
Yi is constructing a cardboard box. The base of the box will be a square of width w inches. The height of the box will be h inches. Yi will use gray cardboard for the sides of the box and brown cardboard for the bottom (the box does not have a top). Gray cardboard costs \$0.05 per square inch, while brown cardboard costs \$0.03 per square inch. Yi wants to spend \$20 on the cardboard for his box.



Write a formula for h in terms of w.

Answer: $h =$ ________________________

8. [7 points] Yi is working on a second box (not the one from the previous problem). This box will need a wire frame. The base of the box will be a square with width w inches. The height of the box will be h inches. Once the box is completed, its volume, V, in cubic inches will be

$$V = w^2 h.$$

Yi has to use thicker wire for the edges along the top and bottom of the box. Let M be the total mass in grams of the wire frame. The equation for M is

$$M = 18w + 6h.$$

Yi has a total of 540 grams of metal to make the wire frame. What values of w and h will maximize the volume of his box? Use calculus to find and justify your answer, and be sure to show enough evidence that the values you find do in fact maximize the volume.

Answer: The volume of the box is maximum when

$w =$ ________________ $h =$ ________________

9. [13 points] A curve $\mathcal{C}$ is defined implicitly by the equation

$$2x(y^2 - 4y) + 5x = 15.$$

Note that the curve $\mathcal{C}$ satisfies

$$\frac{dy}{dx} = \frac{-2y^2 + 8y - 5}{4xy - 8x}.$$

a. [4 points] Find all points on $\mathcal{C}$ with a vertical tangent line. Give your answers as ordered pairs (coordinates). Justify your answer algebraically. Write NONE if no such points exist.

Answer: ______________________________

b. [4 points] There is one point on $\mathcal{C}$ with a coordinate $(k, 0)$. Find the value of k and write the equation of the tangent line to $\mathcal{C}$ at the point $(k, 0)$. Your equation should not include the letter k.

Answer: $k =$ ____________ Equation of tangent line: ____________________

c. [5 points] Another curve $\mathcal{D}$ is defined implicitly by the equation

$$\cos(Wy^3 + Vxy) = \frac{1}{6}$$

where W and V are constants. Find a formula for $\frac{dy}{dx}$ in terms of x, y, W, and V. To earn credit for this problem, you must compute this by hand and show <u>every</u> step of your work clearly.

Answer: $\dfrac{dy}{dx} =$ ________________________________

10. [7 points] The function g has the property that $g(x)$, $g'(x)$, and $g''(x)$ are defined for all real numbers. The quadratic approximation of $g(x)$ at $x = -2$ is

$$Q(x) = 4(x+2)^2 + \frac{1}{2}(x+2) - 5.$$

a. [5 points] Find the exact value of each of the following quantities. If there is not enough information to answer the question, write NI.

$g(-2) =$ ________ $g'(-2) =$ ________ $g''(-2) =$ ________

$g(0) =$ ________ $Q'(0) =$ ________

b. [2 points] Write a formula for $L(x)$, the tangent approximation of $g(x)$ near $x = -2$.

Answer: $L(x) =$ ________________________

11. [8 points] Consider the function

$$f(x) = \begin{cases} 4 - x - x^{\frac{2}{3}} & \text{for} \quad -8 \le x \le 0 \\ 5xe^{-0.5x} + 4 & \text{for} \quad x > 0. \end{cases}$$

and its derivative

$$f'(x) = \begin{cases} \dfrac{2 + 3x^{\frac{1}{3}}}{-3x^{\frac{1}{3}}} & \text{for} \quad -8 < x < 0 \\ 5(1 - 0.5x)e^{-0.5x} & \text{for} \quad x > 0. \end{cases}$$

Find the x-coordinates of the global maximum and the global minimum of the function $f(x)$ for $x \ge -8$. If one of them does not exist, write NONE in the answer line below. Use calculus to find your answers, and be sure to show enough evidence that the point(s) you find are indeed global extrema.

Answer:

Global maximum(s) at $x =$ ____________________

Global minimum(s) at $x =$ ____________________