Math 115 — Second Midterm — November 11, 2019

EXAM SOLUTIONS

1. **Do not open this exam until you are told to do so.**
2. **Do not write your name anywhere on this exam.**
3. This exam has 11 pages including this cover. There are 11 problems.
   Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
4. Do not separate the pages of this exam. If they do become separated, write your UMID (not name) on every page and point this out to your instructor when you hand in the exam.
5. Note that the back of every page of the exam is blank, and, if needed, you may use this space for scratchwork. Clearly identify any of this work that you would like to have graded.
6. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
7. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it.
8. The use of any networked device while working on this exam is not permitted.
9. You may use any one calculator that does not have an internet or data connection except a TI-92 (or other calculator with a "qwerty" keypad). However, you must show work for any calculation which we have learned how to do in this course.
   You are also allowed two sides of a single $3'' \times 5''$ notecard.
10. For any graph or table that you use to find an answer, be sure to sketch the graph or write out the entries of the table. In either case, include an explanation of how you used the graph or table to find the answer.
11. Include units in your answer where that is appropriate.
12. Problems may ask for answers in *exact form*. Recall that $x = \sqrt{2}$ is a solution in exact form to the equation $x^2 = 2$, but $x = 1.41421356237$ is not.
13. **Turn off all cell phones, smartphones, and other electronic devices**, and remove all headphones, earbuds, and smartwatches. Put all of these items away.
14. You must use the methods learned in this course to solve all problems.

Problem	Points	Score
1	13	
2	9	
3	12	
4	13	
5	7	
6	6	

Problem	Points	Score
7	11	
8	11	
9	6	
10	4	
11	8	
Total	100	

1. [13 points] Let g be a function such that $g''(x)$ is defined for all real numbers. A table of values of $g'(x)$, **the derivative of** $g(x)$, is given below.

x	-5	-1	0	3	4	7
$g'(x)$	3	0	-4	-3	0	2

Assume that between each pair of consecutive values of x given in the table, $g'(x)$ is either **always increasing** or **always decreasing**.

For parts **a.–f.**, circle **all** correct choices.

a. [1 point] At which of the following values does $g(x)$ have a critical point?

$x=-5$ $\quad$ $\boxed{x=-1}$ $\quad$ $x=0$ $\quad$ $x=3$ $\quad$ $\boxed{x=4}$ $\quad$ $x=7$ $\quad$ NONE OF THESE

b. [2 points] On which of the following intervals is $g(x)$ always decreasing?

$(-5,-1)$ $\quad$ $\boxed{(-1,0)}$ $\quad$ $\boxed{(0,3)}$ $\quad$ $\boxed{(3,4)}$ $\quad$ $(4,7)$ $\quad$ NONE OF THESE

c. [2 points] At which of the following values does $g(x)$ have a local maximum?

$\boxed{x=-1}$ $\quad$ $x=0$ $\quad$ $x=3$ $\quad$ $x=4$ $\quad$ NONE OF THESE

d. [2 points] On which of the following intervals is $g(x)$ always concave down?

$\boxed{(-5,-1)}$ $\quad$ $\boxed{(-1,0)}$ $\quad$ $(0,3)$ $\quad$ $(3,4)$ $\quad$ $(4,7)$ $\quad$ NONE OF THESE

e. [2 points] At which of the following values does $g(x)$ have an inflection point?

$x=-1$ $\quad$ $\boxed{x=0}$ $\quad$ $x=3$ $\quad$ $x=4$ $\quad$ NONE OF THESE

f. [2 points] Suppose that $g(7)=0$ and $g''(x)<0$ for all $x>7$. Which of the following values of $g(10)$ are possible?

$\boxed{g(10)=-5}$ $\quad$ $\boxed{g(10)=2}$ $\quad$ $g(10)=6$ $\quad$ $g(10)=11$ $\quad$ NONE OF THESE

g. [2 points] Use the table to give the best possible estimate of $g''(-3)$.

Solution: Since -3 is between -5 and -1, we find the average rate of $g'(x)$ on the interval $(-5,-1)$ to estimate the derivative of g' at -3, i.e. $g''(-3)$:

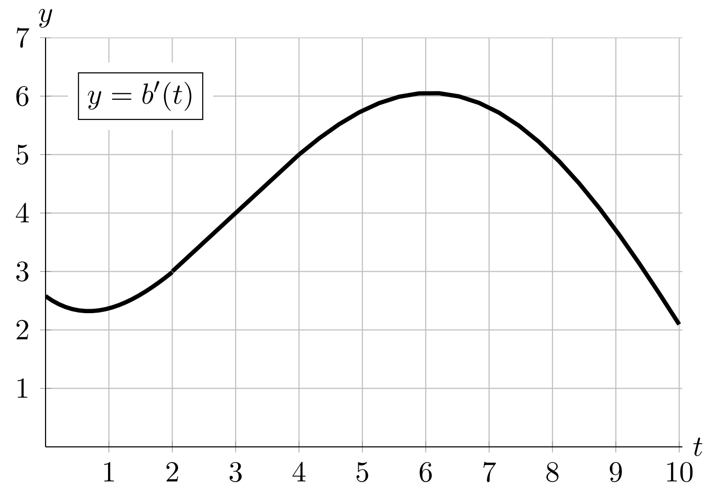
$$\frac{3-0}{-5-(-1)} = \frac{3}{-4}.$$

Answer: $g''(-3) \approx$ $\dfrac{-3}{4}$

2. [9 points]

A scientist conducted an experiment in which she grew a bacterial culture in a petri dish. Let $b(t)$ be the amount of bacteria, as measured by mass in milligrams (mg), contained in the dish t hours into the experiment. A portion of the function $b'(t)$, the **derivative** of $b(t)$, is graphed to the right.

The graph of $b'(t)$ passes through the points $(2,3)$, $(3,4)$, $(4,5)$ and $(8,5)$. You may estimate any other values you need in this problem from the given graph.



a. [2 points] Using the graph, complete the following sentence.

Eight hours into the experiment, in the next ten minutes the amount of bacteria in the dish ...

(circle one) INCREASED DECREASED by approximately 5/6 mg.

Solution: We see that $b'(4) = 5$. Since ten minutes is $1/6$ of an hour, we expect the amount of bacteria to grow by about $5 \cdot 1/6$ mg.

b. [2 points] Four hours into the experiment, there were 32.5 mg of bacteria in the dish. Write a formula for the linear approximation $L(t)$ of $b(t)$ near $t = 4$.

Solution: We know that $L(t) = b(4) + b'(4)(t-4)$. We are told that $b(4) = 32.5$ and see from the graph that $b'(4) = 5$.

Answer: $L(t) =$ $32.5 + 5(t-4)$

c. [2 points] Use $L(t)$ from part **b.** to estimate the amount of bacteria, in mg, in the dish at time $t = 4.3$. Is this estimate an overestimate, an underestimate, neither of these, or is there not enough information to decide?

Solution: We plug in 0.3 to the formula from **b.**. From the graph we see that $b'(t)$ is increasing near $t = 4$, so that $b''(t)$ is positive near $t = 4$. Thus $b(t)$ is concave up near $t = 4$ so this estimate is an underestimate.

Answer: Amount of bacteria at $t = 4.3$ is $\approx$ $32.5 + 5(0.3) = 34$ mg

Circle one: OVERESTIMATE UNDERESTIMATE NEITHER NOT ENOUGH INFO

d. [3 points] Three hours into the experiment, there were 28 mg of bacteria in the dish. Write a formula for the quadratic approximation $Q(t)$ of $b(t)$ near $t = 3$.

Solution: We know that $Q(t) = b(3) + b'(4)(t-3) + \frac{b''(3)}{2}(t-3)^2$. We are told that $b(3) = 28$ and see from the graph that $b'(3) = 4$. We also find the slope of the given graph to find that $b''(3) = 1$.

Answer: $Q(t) =$ $28 + 4(t-3) + \frac{1}{2}(t-3)^2$

3. [12 points] Suppose $q(x)$ is a differentiable function defined for all real numbers x. The <u>derivative</u> and <u>second derivative</u> of $q(x)$ are given by

$$q'(x) = x^{2/3}(x-3)^{5/3}(x+5) \qquad \text{and} \qquad q''(x) = \frac{10(x-3)^{2/3}(x-1)(x+3)}{3x^{1/3}}.$$

a. [1 point] Find the x-coordinates of all critical points of $q(x)$. If there are none, write NONE.

Answer: Critical point(s) of $q(x)$ at $x =$ $x = 0$, 3, **and** -5

b. [2 points] Find the x-coordinates of all critical points of $q'(x)$. If there are none, write NONE.

Answer: Critical point(s) of $q'(x)$ at $x =$ $x = -3$, 0, 1, **and** 3

c. [5 points] Find the x-coordinates of all local maxima and local minima of $q(x)$. If there are none of a particular type, write NONE. Use calculus to find and justify your answers, and be sure to show enough evidence to demonstrate that you have found all local extrema.

Solution: The sign of $q'(x)$ can only possibly change at the three critical points of $q(x)$, namely 0, 3, and -5. Below is a number line showing these signs, with accompanying sign logic in order to justify how we know when $q'(x)$ is positive and when it is negative.
(Note that $a^{2/3} = (a^2)^{1/3}$, which is positive for any nonzero number a and that the sign of $a^{5/3}$ is the same as the sign of a.)

$q'(x)$

Interval	$(-\infty, -5)$	$(-5, 0)$	$(0, 3)$	$(3, +\infty)$
Sign logic	$(-)^{2/3}(-)^{5/3}(-)$	$(-)^{2/3}(-)^{5/3}(+)$	$(+)^{2/3}(-)^{5/3}(+)$	$(+)^{2/3}(+)^{5/3}(+)$
Sign	$(+)$	$(-)$	$(-)$	$(+)$

By the First Derivative Test, $q(z)$ has a local maximum at $x = -5$ and a local minimum at $x = 3$.

Note: The Second Derivative Test can be used to show that $q(x)$ has a local maximum at $x = -5$ but cannot be used to determine the bahavior of $q(x)$ at $x = 0$ or $x = 3$.

Answer: Local max(es) at $x =$ -5 and Local min(s) at $x =$ 3

d. [4 points] Find the x-coordinates of all inflection points of $q(x)$. If there are none, write NONE. Use calculus to find and justify your answers, and be sure to show enough evidence to demonstrate that you have found all inflection points.

Solution: The concavity of $q(x)$ can only possibly change at $x = -3$, 0, 1 and 3. Below is a number line showing the signs of $q''(x)$, which shows when the sign of $q''(x)$ actually changes, with accompanying sign logic in order to justify how we know when $q''(x)$ is positive and when it is negative.

$q''(x)$

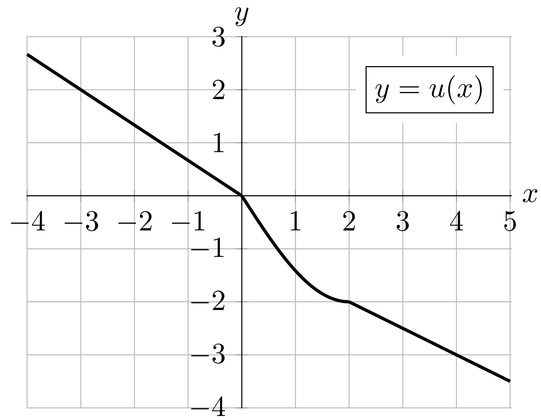
Interval	$(-\infty, -3)$	$(-3, 0)$	$(0, 1)$	$(1, 3)$	$(3, +\infty)$
Sign logic	$\frac{(-)^{2/3}(-)(-)}{(-)^{1/3}}$	$\frac{(-)^{2/3}(-)(+)}{(-)^{1/3}}$	$\frac{(-)^{2/3}(-)(+)}{(+)^{1/3}}$	$\frac{(-)^{2/3}(+)(+)}{(+)^{1/3}}$	$\frac{(+)^{2/3}(+)(+)}{(+)^{1/3}}$
Sign	$(-)$	$(+)$	$(-)$	$(+)$	$(+)$

The sign of $q''(x)$ (and hence the concavity of $q(x)$) changes at $x = -3$, 0, and 1.

Answer: Inflection point(s) at $x =$ -3, 0, **and** 1

4. [13 points]

The function $u(x)$ is defined and invertible on $(-\infty, \infty)$. A portion of its graph is shown to the right.



Note that:

- $u(x) = -2\sin\left(\frac{\pi}{4}x\right)$ on $[0, 2]$, and
- $u(x)$ is linear on the intervals $(-4, 0)$ and $(2, 5)$.

a. [11 points] Evaluate each of the following quantities **exactly**, or write DNE if the value does not exist. You do not need to show work, but limited partial credit may be awarded for work shown. Your answers should not contain the letter u, but do not need to be fully simplified.

i. [2 points] Find $(u^{-1})'(-3)$.

Solution:

$$(u^{-1})'(-3) = \frac{1}{u'(u^{-1}(-3))} = \frac{1}{u'(4)} = \frac{1}{-1/2} = -2$$

Answer: $(u^{-1})'(-3) =$ -2

ii. [2 points] Let $v(x) = u(-1 - x)$. Find $v'(-1)$.

Solution: The graph of $v(x) = u(-x - 1)$ is the result of shifting the graph of $u(x)$ to the right 1 unit and then reflecting it across the y-axis. The point at $x = -1$ on the graph of $v(x)$ comes from the point $(0, 0)$ on the graph of $y = u(x)$, so there is a sharp corner at this point and $v'(-1)$ therefore does not exist.

Answer: $v'(-1) =$ **DNE**

iii. [3 points] Let $w(x) = \dfrac{x}{2^{u(x)}}$. Find $w'(-3)$.

Solution: Note that $u(-3) = 2$ and $u'(-3) = -2/3$.

$$w'(x) = \frac{2^{u(x)} - x\ln(2)\, 2^{u(x)}u'(x)}{\left(2^{u(x)}\right)^2} \quad \text{so} \quad w'(-3) = \frac{2^2 - (-3)\ln(2)\, 2^2(-\frac{2}{3})}{(2^2)^2}$$

Answer: $w'(-3) = \dfrac{4 - 8\ln(2)}{16} = \dfrac{1 - 2\ln(2)}{4}$

iv. [4 points] Let $z(x) = \ln(2x + 1)u(x)$. Find $z'(1)$.

Solution: $u'(x) = -2\frac{\pi}{4}\cos(\frac{\pi}{4}x)$ on $(0, 2)$, so $u(1) = -\sqrt{2}$ and $u'(1) = -\frac{\pi\sqrt{2}}{4}$

$$z'(x) = \ln(2x+1)u'(x) + u(x)\left(\frac{2}{2x+1}\right) \quad \text{so} \quad z'(1) = \ln(3)u'(1) + u(1)\left(\frac{2}{2+1}\right)$$

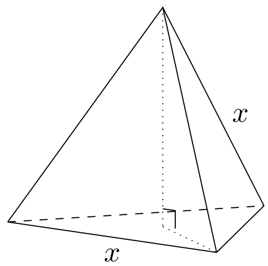
Answer: $z'(1) = \ln(3)\left(\dfrac{-\pi\sqrt{2}}{4}\right) - \sqrt{2}\left(\dfrac{2}{3}\right)$

b. [2 points] At $x = 7$, the tangent line to $u(x)$ is given by $y = -5 - 2(x - 7)$. Find an equation for the tangent line to $u^{-1}(x)$ at $x = -5$.

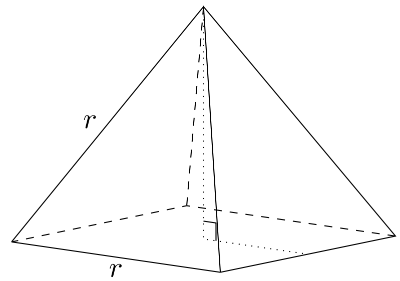
Solution: Since $(u^{-1})'(-5) = \frac{1}{u'(7)} = \frac{1}{-2}$, such an equation is $y = 7 - \frac{1}{2}(x + 5)$.

Answer: $y = 7 - \dfrac{1}{2}(x + 5)$

5. [7 points] An alien is building the wire frames of two pyramids. One has a base that is an equilateral triangle with side length x meters, and the other has a base that is a square with side length r meters. These shapes are shown below. For both, all triangular faces are equilateral.



Triangular Pyramid



Square Pyramid

The alien has 2 meters of wire available to build the frames, and **will use all of it**.

a. [2 points] Find a formula for r in terms of x.

Solution: There are 6 sides of length x meters, and 8 sides of length r meters. In total, these lengths must add up to 2 meters, so $6x + 8r = 2$. We can then solve for r in terms of x and find that $r = \frac{2-6x}{8}$.

Answer: $r = \dfrac{2-6x}{8}$

b. [3 points] Find a formula for $A(x)$, the combined surface area of the two pyramids (i.e. the total area of all sides and bases of both shapes). Your formula should be in terms of x only.

Recall that the area of an equilateral triangle with side length L is $\dfrac{\sqrt{3}}{4}L^2$.

Solution: On the triangular pyramid, there are 4 equilateral triangles each with side length x. On the square pyramid, there are 4 equilateral triangles each with side length r, plus one square of side length r on the base. Adding up the areas of these shapes, we find

$$\text{Total Surface Area} = 4\left(\frac{\sqrt{3}}{4}x^2\right) + 4\left(\frac{\sqrt{3}}{4}r^2\right) + r^2 = \sqrt{3}x^2 + \sqrt{3}r^2 + r^2 = \sqrt{3}x^2 + (\sqrt{3}+1)r^2.$$

We substitute $r = \dfrac{2-6x}{8}$ and find $A(x) = \sqrt{3}x^2 + \left(\sqrt{3}+1\right)\left(\frac{2-6x}{8}\right)^2$

Answer: $A(x) = \sqrt{3}x^2 + \left(\sqrt{3}+1\right)\left(\dfrac{2-6x}{8}\right)^2$

c. [2 points] The alien wants to actually build one of each type of pyramid. In the context of the problem, what is the domain of the function $A(x)$ from part **b.**? You may give your answer as an interval or using inequalities.

Solution: We must have $x > 0$ in order to get a triangular pyramid. We also need $r > 0$ to get a square pyramid, which in terms of x means

$$\frac{2-6x}{8} > 0 \quad \text{which simplifies to} \quad 2 > 6x, \quad \text{so} \quad \frac{1}{3} > x$$

Answer: $\left(0,\ \dfrac{1}{3}\right)$

6. [6 points] Let $K(x)$ be the concentration of krypton gas, in parts per million, at a height of x miles above the surface of a certain alien planet. Formulas for $K(x)$ and $K'(x)$ are given below.

$$K(x) = \frac{(4x-2)^4}{e^{(x+0.5)^2}} \qquad \text{and} \qquad K'(x) = \frac{-16(2x-1)^3(2x-3)(2x+3)}{e^{(x+0.5)^2}}$$

For $x \geq 0$, find the heights at which the concentration of krypton is the smallest and the largest. You must use calculus to find your answers, and be sure to show enough evidence to fully justify your answers. For each answer blank, write NONE if appropriate.

Solution: The critical points of $K(x)$ are $x = -1.5$, 0.5, and 1.5. Since we are optimizing on the interval $[0, \infty)$, we will exclude -1.5 from consideration. Aside from the critical points, we also need to consider the endpoint $x = 0$ and the limit as $x \to \infty$.

$K(0)$	≈ 12.461
$K(0.5)$	0
$K(1.5)$	≈ 4.689
$\lim_{x\to\infty} K(x)$	0

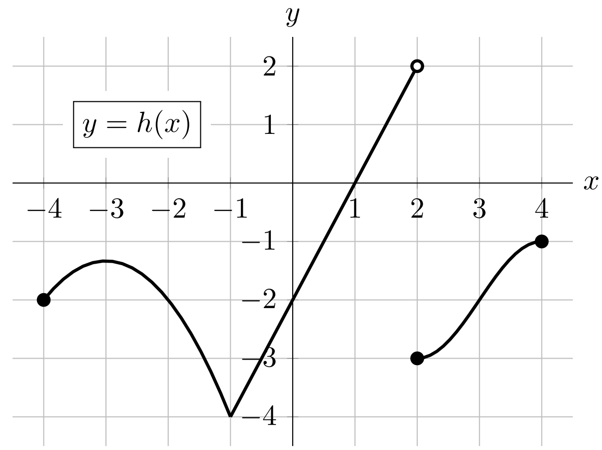
The largest output value is ≈ 12.461, so $K(x)$ attains its global maximum at $x = 0$, i.e. the largest concentration of krypton is found at a height of 0 miles above the the surface.

The smallest value in the table is 0. Because there are no critical points greater than 1.5, we know that the function $K(x)$ must be decreasing on the interval $[1.5, \infty)$, starting at a value of ≈ 4.689, and decreasing towards (but never exactly achieving) a value of 0 on this interval. However, a value of 0 is actually achieved at $x = 0.5$, so $K(x)$ attains its global minimum at $x = 0.5$, i.e. the smallest concentration of krypton is found at a height of 0.5 miles above the the surface.

Answer: Smallest concentration at $x =$ 0.5 **miles**

Largest concentration at $x =$ 0 **miles**

7. [11 points]
Shown to the right is the graph of a function $h(x)$.



For parts **a.**–**c.**, circle **all** correct choices.

a. [2 points] Which of the following are critical points of $h(x)$?

$\boxed{x=-3}$ $\boxed{x=-1}$ $x=1$ $\boxed{x=2}$ $x=3$ NONE OF THESE

b. [2 points] On which of the following interval(s) does $h(x)$ satisfy the hypotheses of the Mean Value Theorem?

$\boxed{[-4,-1]}$ $[-4,0]$ $[0,2]$ $\boxed{[3,4]}$ NONE OF THESE

c. [2 points] On which of the following interval(s) does $h(x)$ satisfy the conclusion of the Mean Value Theorem?

$\boxed{[-4,-1]}$ $\boxed{[-4,0]}$ $[0,2]$ $\boxed{[3,4]}$ NONE OF THESE

d. [5 points] Define the function $k(x)$ such that

$$k(x) = \begin{cases} h(x) & -4 \le x < 1 \\ A^2 \sin(Ax+B) & 1 \le x \le 4, \end{cases}$$

where A and B are constants. Find one pair of values for A and B that make $k(x)$ differentiable at $x = 1$. *Show your work.*

Solution: For k to be differentiable, it must be continuous. At $x = 1$, continuity implies that

$$0 = A^2 \sin(A+B), \quad \text{so} \quad A = 0 \text{ or } \sin(A+B) = 0.$$

We also need the slope of each piece to match at $x = 1$, that is,

$$h'(1) = A^2\left(\cos(A+B)\cdot A\right), \quad \text{so} \quad 2 = A^3\cos(A+B).$$

Notice that we can rule out the possibility that $A = 0$ (since $2 \neq 0$), which forces us to choose $\sin(A+B) = 0$. The problem only asks for *one* pair of values, and *one* way to get $\sin(A+B) = 0$ is to set $A+B = 0$ so $A = -B$. This means

$$2 = A^3\cos(0) = A^3, \quad \text{which we can solve to find} \quad A = \sqrt[3]{2} \quad \text{and} \quad B = -A = -\sqrt[3]{2}.$$

Note: There are other possible values. We could have also chosen $A + B = n\pi$ where n is any integer. If n is even, this gives $(A,B) = \left(\sqrt[3]{2}, n\pi - \sqrt[3]{2}\right)$. If n is odd, this gives $(A,B) = \left(-\sqrt[3]{2}, n\pi + \sqrt[3]{2}\right)$.

Answer: $A =$ $\sqrt[3]{2}$ and $B =$ $-\sqrt[3]{2}$

8. [11 points] Parts **a.** and **b.** are unrelated.

a. [6 points] Windchill is the temperature felt on exposed skin due to the combination of air temperature and wind speed. For a certain fixed air temperature, we define the following functions W and T.

- $W(s)$ is the windchill, in degrees Fahrenheit, when the wind speed is s miles per hour (mph).
- $T(r)$ is the time, in minutes, it takes for frostbite to develop on exposed skin when the windchill is r degrees Fahrenheit.

The functions W and T are both invertible and differentiable. Suppose that

- $W(25) = -37$
- $W'(25) = -0.4$
- $T(-25) = 25$
- $T'(-25) = 2$
- $T(-37) = 10$
- $T'(-37) = 0.75$

i. [2 points] Write an equation for the linear approximation $L(s)$ of $W(s)$ near $s = 25$.

Answer: $L(s) =$ $-37 - 0.4(s - 25)$

ii. [1 points] How many minutes does it take for frostbite to develop if the wind speed is 25 mph?

Answer: 10

iii. [3 points] If the wind speed is 26 mph, estimate the amount of time, in minutes, it takes for frostbite to develop.

Solution: To estimate $T(W(26))$, one option is to use the linear approximation of $T(W(s))$ near $s = 25$. By the chain rule, $\frac{d}{ds}(T(W(s)) = T'(W(s))W'(s)$, so this linear approximation is

$$\begin{aligned} T(W(25)) + T'(W(25))W'(25)(s-25) &= 10 + (0.75)(-0.4)(s-25) \\ &= 10 - 0.3(s-25). \end{aligned}$$

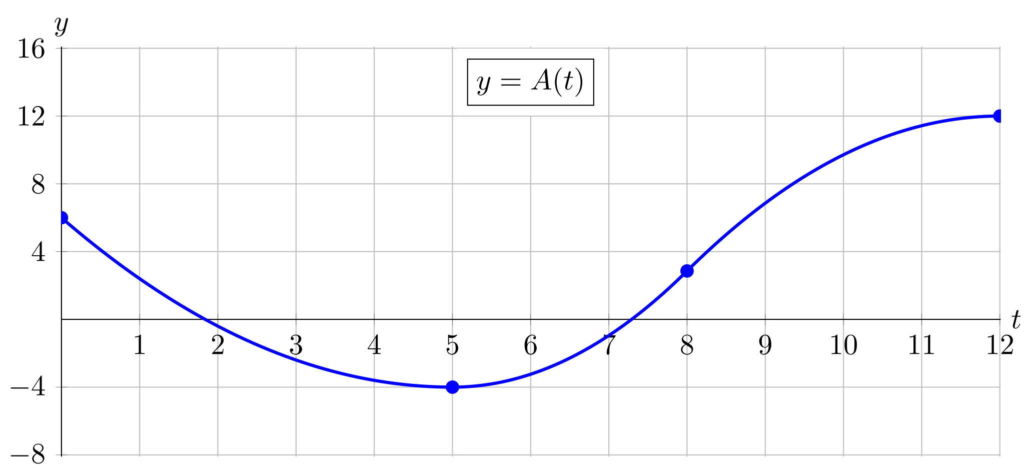
So $T(W(26)) \approx 10 - 0.3(26 - 25) = 9.7$.

Answer: 9.7

b. [5 points] Let $A(t)$ be the temperature, in degrees Fahrenheit (°F), at time t hours after midnight on a certain winter day in Ann Arbor. You are given the following information.

- $A(t)$ is differentiable and has only one critical point on $0 < t < 12$.
- The coldest temperature that day was -4°F, which occurred at 5:00 AM.
- Between midnight and 5:00 AM, the temperature fell at an average rate of 2°F per hour.
- The temperature was increasing the fastest at 8:00 AM.
- The global maximum value of $A(t)$ on $0 \le t \le 12$ is 12°F.

On the axes below, sketch a possible graph of $A(t)$ on $0 \le t \le 12$.



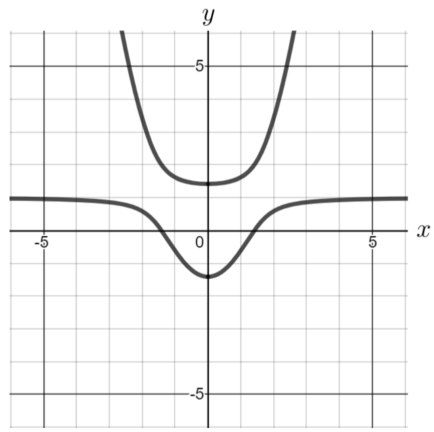
9. [6 points]
The implicit curve $\mathcal{C}$ is given by the equation

$$y^2 - 1 = r^2 + x^2(y - r)$$

for some constant r. A graph of the curve with $r = 1$ is shown to the right. Note that

$$\frac{dy}{dx} = \frac{2x(y-r)}{2y - x^2}.$$

Answer each of the following questions about the implicit curve $\mathcal{C}$. Your answers must be in **exact form**.



a. [2 points] When $r = 1$, the curve $\mathcal{C}$ passes through the point $(\sqrt{2}, 0)$. Write a formula for the tangent line to the curve $\mathcal{C}$ at this point.

Solution: The slope at $(\sqrt{2}, 0)$ when $r = 1$ is

$$\frac{2\sqrt{2}(0-1)}{2(0) - (\sqrt{2})^2} = \frac{2\sqrt{2}}{2} = \sqrt{2}.$$

Answer: $y = \sqrt{2}\,(x - \sqrt{2})$

b. [4 points] In this part, we do not assume anything about r. In particular, do <u>not</u> assume $r = 1$. Find the (x, y) coordinates of **<u>all</u>** points at which the tangent line to the curve $\mathcal{C}$ is horizontal. If there are no such points, write NONE. Your answer may be in terms of the constant r. You must show every step of your work.

Solution: For the tangent line to $\mathcal{C}$ at (x, y) to be horizontal, we need the numerator of dy/dx to equal zero:

$$2x(y - r) = 0, \quad \text{meaning} \quad x = 0 \quad \text{or} \quad y = r.$$

If there was a point on the curve with $y = r$, then we would have the equation

$$\begin{aligned} r^2 - 1 &= r^2 + x^2(r - r) \\ r^2 - 1 &= r^2 \\ -1 &= 0, \end{aligned}$$

so there are no points with $y = r$. To find points with $x = 0$, we solve

$$\begin{aligned} y^2 - 1 &= r^2 + (0)^2(y - r) \\ y^2 &= r^2 + 1 \\ y &= \pm\sqrt{r^2 + 1}. \end{aligned}$$

Answer: $\left(0, \sqrt{r^2+1}\right)$ **and** $\left(0, -\sqrt{r^2+1}\right)$

10. [4 points] An implicit curve is described by the equation

$$xy^n = \cos(ax)$$

where a and n are positive constants. Compute $\frac{dy}{dx}$. Your answer may include a and n. You must show every step of your work.

Solution:

$$\frac{d}{dx}(xy^n) = \frac{d}{dx}(\cos(ax))$$

$$x \cdot \left(ny^{n-1}\frac{dy}{dx}\right) + 1 \cdot y^n = -a\sin(ax)$$

$$\left(nxy^{n-1}\right)\frac{dy}{dx} = -a\sin(x) - y^n$$

$$\frac{dy}{dx} = \frac{-a\sin(ax) - y^n}{nxy^{n-1}}$$

Answer: $\frac{dy}{dx} = \frac{-a\sin(ax) - y^n}{nxy^{n-1}}$

11. [8 points] The differentiable function $f(x)$ is defined for all real numbers. Additionally, $f(x)$ has **exactly two** critical points, at $x = 0$ and $x = 5$. A table of values of $f(x)$ is given below.

x	-2	1	3	7
$f(x)$	2	4	9	5

For parts **a.–d.**, circle **all** correct choices.

a. [2 points] On which of the following interval(s) must $f'(x)$ always be negative?

$(-2, 0)$ $(0, 1)$ $(1, 5)$ **[$(5, 7)$]** NONE OF THESE

b. [2 points] On which of the following interval(s) must there be a point c for which $f'(c) = -1$?

$(-\infty, -2)$ $(-2, 1)$ $(1, 3)$ **[$(3, 7)$]** NONE OF THESE

c. [2 points] On the interval $[0, 6]$, at which of the following point(s) does $f(x)$ attain its global maximum? If there is not enough information to determine this, circle NOT ENOUGH INFO.

$x = 0$ $x = 1$ **[$x = 5$]** $x = 6$ NOT ENOUGH INFO

d. [2 points] On the interval $[-2, 5]$, at which of the following point(s) does $f(x)$ attain its global minimum? If there is not enough information to determine this, circle NOT ENOUGH INFO.

$x = -2$ $x = 0$ $x = 2$ $x = 5$ **[NOT ENOUGH INFO]**