Math 115 — Final Exam — December 12, 2025

Write your 8-digit UMID number very clearly in the box to the right.

Your Initials Only: ______ Instructor Name: ________________________ Section #: __________

1. Please neatly write your 8-digit UMID number, your initials, your instructor's first and/or last name, and your section number in the spaces provided.
2. This exam has 11 pages including this cover.
3. There are 9 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it.
6. If you need more space to answer a question, please use the back of an exam page. Clearly indicate on your exam if you are using the back of a page, and also clearly label the problem number and part you are doing on the back of the page.
7. You are allowed notes written on two sides of a $3'' \times 5''$ note card. You are NOT allowed other resources, including, but not limited to, notes, calculators or other electronic devices.
8. For any graph or table that you use to find an answer, be sure to sketch the graph or write out the entries of the table. In either case, include an explanation of how you used the graph or table to find the answer.
9. Include units in your answer where that is appropriate.
10. Problems may ask for answers in *exact form*. Recall that $x = \sqrt{2}$ is a solution in exact form to the equation $x^2 = 2$, but $x = 1.41421356237$ is not.
11. You must use the methods learned in this course to solve all problems.

Problem	Points	Score
1	12	
2	8	
3	6	
4	14	
5	8	

Problem	Points	Score
6	9	
7	14	
8	9	
9	10	
Total	90	

1. [12 points] You are moving straight upward in a hot air balloon.

- Your altitude $a = h(t)$, in miles, is a function of your time t, in hours, since takeoff.
- The air temperature $T = m(a)$ outside your balloon, in degrees Fahrenheit (°F), is a function of your altitude a, in miles.

Both h and m are differentiable and invertible. The following values are known.

- $h(0.5) = 0.33$
- $h'(0.5) = 2$
- $m(0.33) = 55$
- $m'(0.33) = -16$
- $h(1) = 0.5$
- $h'(1) = 0.33$
- $m(0.5) = 52.8$
- $m'(0.5) = -15$

a. [5 points] For each of the following two equations: fill in the missing value, then use a complete sentence to interpret the equation practically.

$m(h(0.5)) =$ ________

Interpretation:

$\int_{0.5}^{1} h'(t)\, dt =$ ________

Interpretation:

b. [3 points] Recall that $m(0.33) = 55$ and $m'(0.33) = -16$. Use these two values to estimate the temperature at an altitude of 0.43 miles. Include units and show your work.

Answer (include units): $\approx$ ____________

c. [4 points] How fast is the air temperature outside your balloon decreasing 30 minutes after takeoff? That is, at what rate is the air temperature decreasing as a function of **time** at this instant? Include units, and show your work to justify your answer.

Answer (include units): ________________

2. [8 points] Consider the family of functions given by $h(x) = ax^{2/3} + bx$ with parameters a and b. It may be helpful to recall that $8^{1/3} = 2$ and $64^{1/3} = 4$.

a. [4 points] Find values of a and b so that $h(x)$ has a critical point at $(8, 4)$.

Answer: $a =$ ____________ $b =$ ____________

b. [4 points] Given the values you of a and b that you found in the previous part, classify all local extrema of $h(x)$. If there aren't any local extrema of a particular type, write NONE. **Be sure you show enough evidence** to support your conclusions.

Answer: Local Min(s): $x =$ ______________ Local Max(es): $x =$ ______________

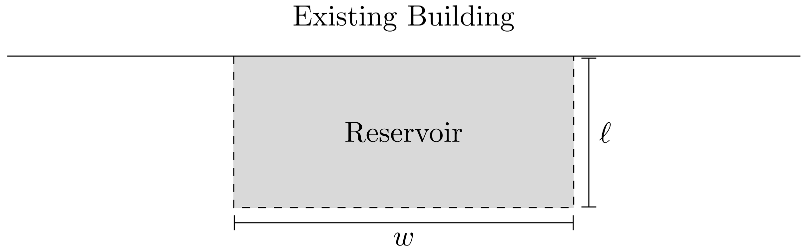
3. [6 points] Sarah is making a bowl on a pottery wheel with a fixed amount of clay. The height of the bowl and the thickness of the walls are related by the following equation:

$$25 = w^2(h^2 + 9),$$

where h is the height of the bowl in inches and w is the thickness of the walls of the bowl in inches. When the bowl is 4 inches tall, the height is increasing at a rate of 0.5 inches per second. How fast is the thickness of the walls changing? Is the thickness increasing or decreasing? **Include units.**

Answer: (circle one) INCREASING DECREASING at a rate of ____________________

4. [14 points]
A city is building a new water treatment facility. A water reservoir will be built next to an existing building as shown here (viewed from above):



Three walls of the reservoir (dashed) and the floor of the reservoir (shaded) will be made of concrete. The cost for the concrete on the three walls is \$500 per meter of perimeter and the cost for the concrete for the floor is \$1000 per square meter.

a. [3 points] Find a formula for the cost C for the concrete to build the reservoir if it has dimensions ℓ and w (both in meters) as shown above.

Answer: $C =$ ______________________________

b. [2 points] If the budget for the total cost of the concrete is \$5,000,000, write a formula for the length ℓ of the reservoir in terms of its width w. This relationship can be used to find the largest reservoir possible, given the concrete budget, but you do not need to find this.

Answer: $\ell =$ ______________________________

4. (continued) The city is also looking for the most cost-effective way to build a large tank in the shape of a cylinder for the water treatment facility. The cost, in dollars, of a tank with radius r and height h (both given in meters) is given by

$$50\pi r^2 + 80\pi rh.$$

c. [2 points] The tank must have volume 10,000 cubic meters, so that $\pi r^2 h =$10,000. Given this constraint, find a formula for the cost $T(r)$ of the tank, in dollars, that is a function of the variable r only. *Your answer should not include the height h of the tank.*

Answer: $T(r) =$ ________________________

d. [2 points] Note that, in context, $T(r)$ has a domain of $(0, \infty)$. Determine $\lim_{r \to 0^+} T(r)$ and $\lim_{r \to \infty} T(r)$. Each answer should either be a number, or ∞ or $-\infty$.

$\lim_{r \to 0^+} T(r) =$ __________ $\lim_{r \to \infty} T(r) =$ __________

e. [5 points] Find the radius that will minimize the cost of the tank. Use calculus, and show your work, but you need not simplify your numerical answers. *Make sure to justify why your answer is the global minimum; you may use work from previous parts of this problem.*

Answer: Cost is minimized when $r =$ ____________

5. [8 points] Given below is a table of values for the first and second derivatives of a function $p(t)$. The values of $p'(t)$ and $p''(t)$ exist for all real numbers t. Between any two consecutive values of t in the table, each of $p'(t)$ and $p''(t)$ is either **always positive** or **always negative**.

t	-1	1	3	5
$p'(t)$	0	-4	-2	-1
$p''(t)$	-3	0	0	2

a. [2 points] Estimate $p'''(0)$. Show your work to justify your answer.

Answer: $p'''(0) \approx$ ______________

For the following parts, you do not need to show any work.

b. [2 points] Which of the following ***must*** be a critical point of $p(t)$? Circle all that apply.

$t = -1$ $\qquad$ $t = 1$ $\qquad$ $t = 3$ $\qquad$ $t = 5$ $\qquad$ None of these

c. [2 points] At which of the following ***could*** $p'(t)$ have a local maximum? Circle all that apply.

$t = -1$ $\qquad$ $t = 1$ $\qquad$ $t = 3$ $\qquad$ $t = 5$ $\qquad$ None of these

d. [2 points] At which of the following ***could*** $p(t)$ have an inflection point? Circle all that apply.

$t = -1$ $\qquad$ $t = 1$ $\qquad$ $t = 3$ $\qquad$ $t = 5$ $\qquad$ None of these

6. [9 points] Clementine wants to make some strawberry jam and sell it to the people in their community. The cost for Clementine to produce q jars of jam is given by the function

$$C(q) = 12\sqrt{q+1} + 3q - 2.$$

Clementine can produce up to 35 jars of jam. Clementine can sell the first 24 jars for \$6. After that, they will sell the remaining jars to their friends for \$3. The revenue is a **continuous** function given by

$$R(q) = \begin{cases} 6q & 0 \le q \le 24 \\ 3(q-24) + 144 & 24 < q \le 35 \end{cases}$$

a. [1 point] What is Clementine's fixed cost, in dollars?

Answer: ______________________

b. [4 points] For what quantities of jars of jam sold would Clementine's marginal cost equal their marginal revenue? Write NONE if there are no such quantities.

Answer: ______________________

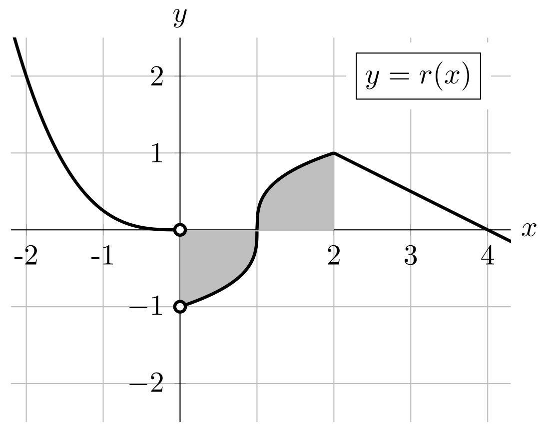
c. [4 points] How many jars of jam should Clementine make to maximize their profit, and what would their maximum profit be? **Be sure you show enough evidence** and support your conclusions using calculus.

Answer: Number of jars of jam: ____________ Maximum profit: ____________

7. [14 points]

A portion of the graph of the function $r(x)$ is shown to the right. Note that:

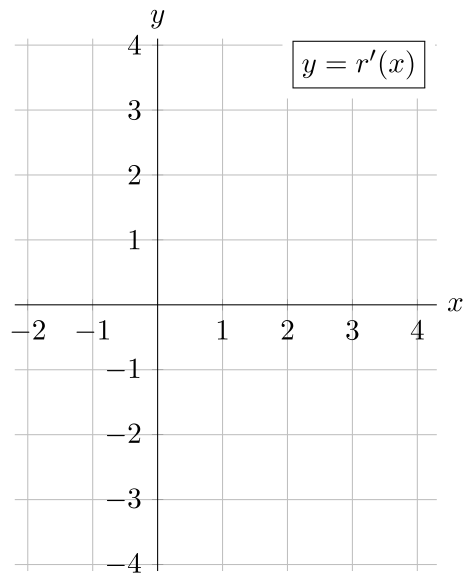
- On the interval $-2 \le x < 0$, the function is given by the formula $-\frac{1}{4}x^3$.
- The two shaded regions each have an area of $\frac{3}{4}$.
- There is a vertical tangent line at $x = 1$ and a corner at $x = 2$.
- The graph is linear on $2 \le x \le 4$.



a. [6 points]

On the axes to the right, sketch a detailed graph of $r'(x)$, the derivative of $r(x)$, for $-2 \le x \le 4$. Make sure that the following are clear from your graph:

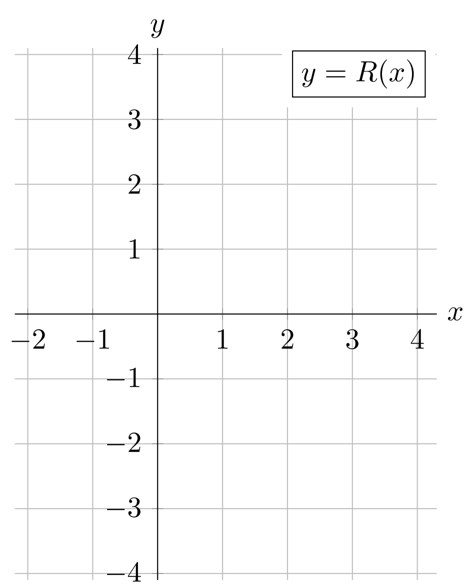
- where $r'(x)$ is undefined
- any vertical asymptotes of $r'(x)$
- where $r'(x)$ is zero, positive, and negative
- where $r'(x)$ is increasing, decreasing, and constant



b. [8 points]

Let $R(x)$ be a continuous antiderivative of $r(x)$ with $R(0) = -1$. On the axes to the right, sketch a detailed graph of $R(x)$ for $-2 \le x \le 4$. Make sure that the following are clear from your graph:

- where $R(x)$ is and is not differentiable
- the values of $R(x)$ at $x = -2$, 0, 1, 2, and 4
- where $R(x)$ is increasing, decreasing, and constant
- where $R(x)$ is linear (with correct slope)
- the concavity of $R(x)$



8. [9 points] You do not need to show any work on this problem.

a. [3 points] The linear approximation near $x = 1$ of a function $g(x)$ is given by $L(x) = 2(x-1)+3$. Which of the following **could** be $g(x)$? Circle all that apply.

$2(x-1)+3$	2^x+1	$\sin(2(x-1))+3$
x^2+1	$-\cos(2(x-1))+3$	NONE

b. [3 points] Which of the following limits are equal to zero? Circle all that apply.

$\lim\limits_{x\to\infty} \frac{x^{1000}}{e^x}$	$\lim\limits_{h\to 0} \frac{\cos(h)-1}{h}$	$\lim\limits_{x\to\infty} \frac{x^2}{\ln x + x}$
$\lim\limits_{x\to\infty} \frac{2^{-x}+3}{3^{-x}+1}$	$\lim\limits_{x\to 0} \frac{x^3+2x}{x^2+1}$	NONE

c. [3 points] Let $r(t)$ be the the number of inches of snow that fell in Ann Arbor since midnight, t hours after midnight on November 30. Which of the following is the BEST practical interpretation of the equation $r'(1) = 10$? Circle the numeral of the **one** best practical interpretation.

(i) Between 1:00am and 2:00am, 10 inches of snow fell in Ann Arbor.

(ii) At 10:00am, snow fell at a rate of 1 inch per hour.

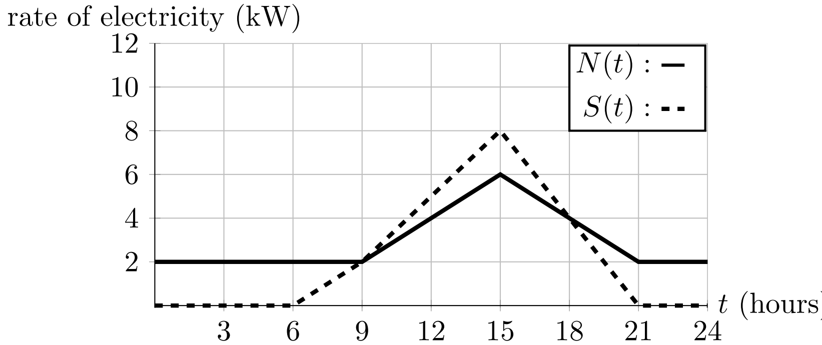
(iii) Between 12:54am and 1:00am, approximately 1 inch of snow fell in Ann Arbor.

(iv) Between 1:00am and 1:10am, approximately 1 inch of snow fell in Ann Arbor.

9. [10 points] Nikki's electric company charges her \$0.20 per kilowatt-hour (kWh) that she uses. Suppose that, during a typical 24-hour period, the rate at which Nikki uses electricity, in kilowatts (kW), t hours after midnight, is given by the function $N(t)$, the solid graph shown below. Note that Nikki's electricity usage over a given time interval is given by the integral of $N(t)$ over that interval.

Nikki is considering installing solar panels. Energy generated by the solar panels can be used free of cost, but when the panels produce more energy than Nikki uses, the energy is sent back into the electric grid, and the electric company pays Nikki \$0.05 per kWh for that energy.

Suppose $S(t)$ is the rate at which the solar panels would generate electricity, in kW, over the same 24-hour period, t hours after midnight. The function $S(t)$ is shown by the dashed graph below.



a. [2 points] Write an expression, using a definite integral involving $S(t)$ or its derivative, for the amount of electricity, in kWh, Nikki's solar panels would generate over this 24-hour period.

Answer: ______________________

b. [2 points] Write an expression, using a definite integral involving $N(t)$ or its derivative, for Nikki's average rate of electricity use, in kW, between 9:00am and 3:00pm. Do not compute the average value.

Answer: ______________________

c. [2 points] How much would Nikki pay the electric company for the energy used between 6:00pm and 9:00pm, if she installed the solar panels? Include units.

Answer: ______________________

d. [4 points] How much would Nikki's electric bill be over this 24-hour period, if she installed the solar panels? The cost of energy supplied by the electric company and used by Nikki is added to the bill, and the amount the electric company pays Nikki is subtracted from the bill. Be sure to clearly show all of your work to receive full credit. Include units.

Answer: ______________________