Math 115---Exam I
Winter 2001

DEPARTMENT of MATHEMATICS
University of Michigan

January 31, 2001

Name: ______________________________ **Signature:** __________________________________

Instructor: __________________________________ **Section No:** _____________

General Instructions: *Do not open this exam until you are told to begin.* This test consists of 8 questions on 7 pages (including this cover sheet). The exam is worth 100 points. Do not separate the exam. If any pages do become detached, write your name on them and point them out to your instructor when you turn in the exam.

Please read the instructions for each individual exercise carefully. Show an appropriate amount of work for each exercise so that graders can see not only the answer but also how you obtained it. If you use graphs or tables to obtain an answer, be certain to provide an explanation (and a sketch of the graph, if that is the method) to make it clear how you arrived at your solution. Use units where appropriate.

Problem No.	Points	Score
1	12	
2	5	
3	12	
4	12	
5	10	
6	15	
7	16	
8	18	
Total	100	

1.) (2 pts each) **True / False--**Circle your choice. Circle **T** only if the statement is always true. [No explanation necessary.]

(a) $\ln(AB) = (\ln A)(\ln B)$ **T** **F**

(b) $\ln e^{(2t-1)} = 2t-1$ **T** **F**

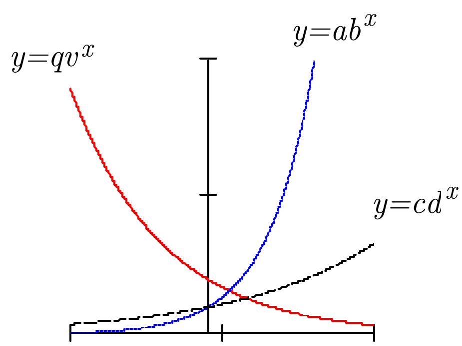
(c) $\sin(3a) = 3\sin(a)$ **T** **F**

(d) As x ® ¥, x^{100} dominates 1.001^x **T** **F**

(e) $\log(10A) = \log A + 1$ $(A > 0)$ **T** **F**

(f) A 5th degree polynomial must have at least one real zero. **T** **F**

2.) (5 pts--No explanation necessary.) The graphs of three functions are given in the figure below.



[**Note: On the original exam, these functions were labeled.**

Complete each of the statements below by using the symbols **>, <,** or **= .**

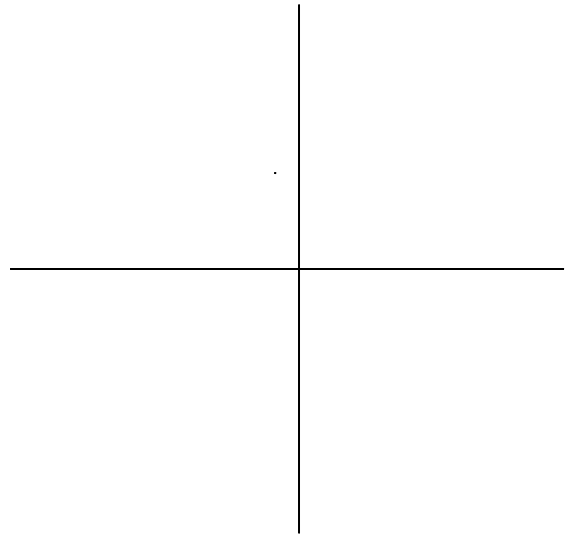
a ____ q a ____ c b ____ d d _____ v

Which, if any, of the parameters $a,b,c,d,q,,v$ are greater than zero?

____

3.) (12 pts) (a) On the axes below, sketch a graph of a single *continuous* function, $y = f(x)$, which has ***all*** of the following features:

- $f(0) = -3$
- $f(-2) = 0$ and $f(3) = 0$
- f is decreasing for $x < 0$
- f is increasing for $x > 0$
- f is concave up for $x < 2$
- f is concave down for $x > 2$
- *f(x)* ® *4* as *x*® ¥



(b) Is the function you drew in part (a) *invertible*? Explain why or why not.

4.) Data from three functions is shown in the table below. One function is linear, one is a power function, and one is neither of these.

x	-2	0	2	4	6	8
$f(x)$	16.5	20	24.2	29.3	35.4	42.9
$g(x)$	17.6	20	22.4	24.8	27.2	29.6
$h(x)$	4.4	0	4.4	17.6	39.6	70.4

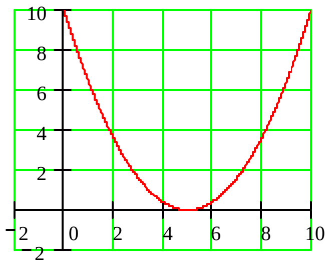
(a) (6 pts) Determine a formula for the linear function. [Be certain to use the appropriate function name—*i.e., f, g,* or *h,* from the table.]

____

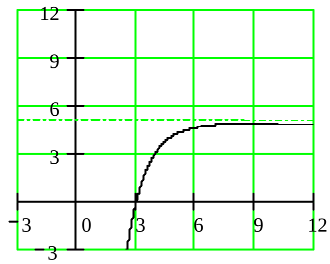
(b) (6pts) Determine a formula for the power function. [Again use the correct function name.]

____

5.) (10 pts) The graphs of f and g are given in the figures below, along with the asymptote to the graph of g.



y = f(x)



y = g(x)

Using the graphs, determine approximate values (to the nearest integer) for each of the following:

(a) $f(g(3))$ __________ (b) $g^{-1}(f(8))$ ___________ (c) $f^{-1}(0)$ ___________

(d) $f(g(1{,}000{,}000))$ ___________ (e) $g(g^{-1}(3))$ ___________

6.) (15 pts) Determine the *zeros* (if any) and describe the behavior as $x \to \infty$ of the following functions: [No explanation necessary.]

(a) $f(x) = \dfrac{5(x+1)(1-x)}{(x+2)(x-3)}$ zeros: _________________

as $x \to \infty$, $f(x) \to$ _________

(b) $g(x) = \dfrac{(x^2+1)}{(x+2)}$ zeros: _________________

as $x \to \infty$, $g(x) \to$ _________

(c) $h(x) = -2x(x-3)(x+4)$ zeros: _________________

as $x \to \infty$, $h(x) \to$ _________

(d) $j(x) = (x-2)^3(3x+1)$ zeros: _________________

as $x \to \infty$, $j(x) \to$ _________

(e) Using the function from part (d), write a formula for $m(x)$, given $m(x) = j(x-1)$. [No need to "expand," but do simplify.]

$m(x) =$ _________________________

7.) The populations of Michigan and Arizona between the years of 1960 and 1990 can be modeled by the following functions, where t is the number of years since 1960, and the units of the population is in millions.

Michigan: $f(t) = 7.8(1.0058)^t$; **Arizona**: $g(t) = 1.3(1.035)^t$

(a) (3 pts) [No sentence necessary.] Over the 30 year period, what was the annual percent growth rate for the population of Arizona?

How much greater was that than the corresponding rate for Michigan?

(b) (2 pts) What was the difference in the two populations in 1960? [No sentence needed.]

(c) (4 pts) If the two states continue to grow according to the patterns given above, will there be a time when the population of Arizona will surpass that of Michigan? If not, explain (mathematically) why not. If so, give the year. [Show your work and express your answer in sentence form.]

(d) (2 pts) How many people would the model predict for the population of Michigan in the 2000 census? [No sentence necessary—show work.]

(e) (2 pts) Interpret, in the context of this problem, the ***meaning*** of $g^{-1}(2)$. [Sentence form, of course.]

(f) (3 pts) According to the model above, in what year was the population of Michigan 5 million people? [Show work and express answer in sentence form.]

8.) **Essay Question.** ***All*** answers should be in complete sentences.

Average daily temperature for any city in the United States can be approximated with reasonable accuracy by a function of the form ***f(t) = A* sin(b(t - *h*)) + *k*** , where *t* is in days after January 1.

For example, a model for average daily temps in the following cities is given by:

Phoenix, AZ:	$f(t) = 20\sin\left(\frac{2p}{365}(t-109)\right)+71$
Honolulu, HI:	$f(t) = 4\sin\left(\frac{2p}{365}(t-141)\right)+75$
Bismarck, ND:	$f(t) = 30\sin\left(\frac{2p}{365}(t-110)\right)+40$

(a) (3 pts) Explain why it is appropriate to use $b = \left(\frac{2p}{365}\right)$.

The average temperature in Pittsburgh can be modeled by the function

$$f(t) = 22\sin\left(\frac{2p}{365}(t-118)\right)+40 \quad .$$

(b) (3 pts) According to this model, what is the highest average temperature in Pittsburgh, and in approximately what month during the year does that occur?

(c) (3 pts) What is the lowest average temperature in Pittsburgh, and in what month does that occur?

[This problem is continued on the next page.]

The model for average daily temperature from the previous page was given as

$$f(t) = A \sin(b(t - h)) + k \, .$$

(d) (3 pts) In this model, what does the parameter A tell you about the prevailing climate in a city?

(e) (3 pts) What is the effect of the parameter h in the context of these models (*i.e.*, in terms of temperature and days)?

(f) (3 pts) What does the parameter k indicate in terms of climate?