Math 115---Exam II
Winter 2001

DEPARTMENT of MATHEMATICS
University of Michigan

March 14, 2001

Name: ______________________________ **Signature:** __________________________________

Instructor: __________________________________ **Section No:** _____________

General Instructions: *Do not open this exam until you are told to begin.* This test consists of 9 questions on 8 pages (including this cover sheet). The exam is worth 100 points. Do not separate the exam. If any pages do become detached, write your name on them and point them out to your instructor when you turn in the exam.

Please read the instructions for each individual exercise carefully. Show an appropriate amount of work for each exercise so that graders can see not only the answer but also how you obtained it. If you use graphs or tables to obtain an answer, be certain to provide an explanation (and a sketch of the graph, if that is the method) to make it clear how you arrived at your solution. Use units where appropriate.

Problem No.	Points	Score
1	10	
2	9	
3	15	
4	10	
5	14	
6	8	
7	12	
8	12	
9	10	
Total	100	

(1.) (2 pts each) **True / False--**Circle your choice. Circle **T** only if the statement is ***always*** true. [No explanation necessary.]

(a) If a function is differentiable, then it is continuous. **T** **F**

(b) If a function is continuous, then it is differentiable. **T** **F**

(c) If $f'(x)$ is increasing, then f is concave up. **T** **F**

(d) If $f''(x) = -3$, then f is decreasing. **T** **F**

(e) If f has a critical point at $x=3$, then f has a local maximum or a local minimum at $x=3$. **T** **F**

(2.) Given :

$r(2) = 2$	and	$s(2) = 1$
$r(4) = -1$		$s(4) = 2$
$r'(2) = 5$		$s'(2) = 3$
$r'(4) = -3$		$s'(4) = 4$

Determine the values indicated below ***or*** state clearly what information is needed (and not supplied) to determine the requested value. In each case, first determine a general formula for the derivative function and then find the requested value (if possible). [Circle your answers.]

(3 pts each) Find:

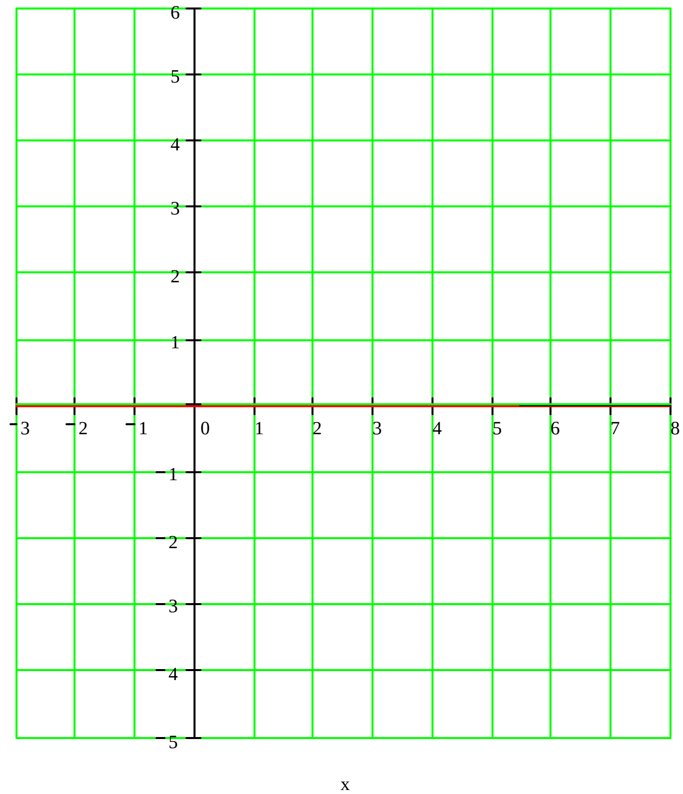
(a) $H'(2)$ if $H(x) = \ln(r(x))$

(b) $H'(2)$ if $H(x) = \dfrac{s(x)}{r(x)}$

(c) $H'(2)$ if $H(x) = \sqrt{s(x)}$

(3.) (6 pts) (a) On the axes below, sketch a graph of a single *differentiable* function, $y = f(x)$, which has ***all*** of the following features:

- *f*(5)= 4
- *f* ¢ *(*5*)* = -1
- *f* ¢ *(x)* > 0 for all *x* < 4
- *f* ¢¢ *(x)* > 0 for all *x* < 2
- *f* ¢¢ *(x)* < 0 for all *x* > 2
- *f* ¢ *(x)* < 0 for all *x* > 4



(b) (4 pts) Using the given information, find an equation of the line tangent to the graph of *f* at $x = 5$.

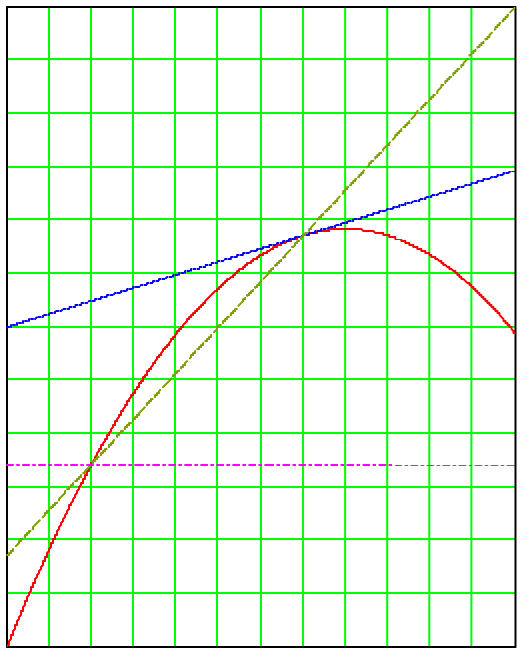
(c) (2 pts) Use your answer from part (b) to approximate *f(*6*)*.

(d) (3 pts) From the ***given*** conditions (*i.e.*, not just from your graph), should the approximation in part (c) be an overestimate or an underestimate? Explain--using a complete sentence.

(4.) Each of the expressions below describes either a ***length*** or a ***slope*** of one of the lines shown in the figure below. In the first blank following each expression, write either the word "length" or "slope," and in the second blank use the letters from the figure to identify the line or line segment. Each line or segment must be identified by **two** letters. For example, if you were given "*h*," the answer would be

h is the _____length______ of the line between _______B & F_______ .

[Note: On the original exam, points on these graphs were labeled A, B, C, ..., etc.]



(10 pts—2 pts each)

(a) *f(a)* is the __________________ of the line between ____________________

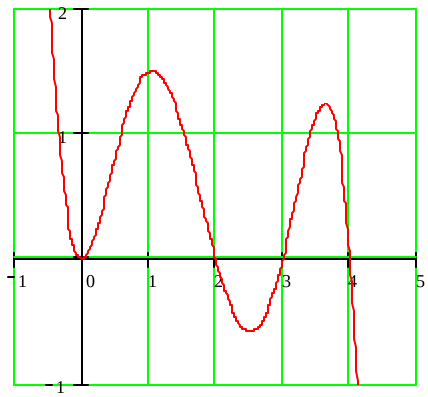
(b) *f(a+h)* is the ________________ of the line between ____________________

(c) *f(a+h) – f(a)* is the __________________ of the line between ____________________

(d) $\dfrac{f(a+h)-f(a)}{h}$ is the ____________________ of the line between ______________

(e) $\lim_{h\to 0} \dfrac{f(a+h)-f(a)}{h}$ is the ______________ of the line between ______________

(5.) (14 pts) The graph in the figure below is the graph of *f ¢ (x)* (*i.e.*, the graph of the ***derivative*** of *f*). [Note: all questions refer to *f* , not *f ¢* .]



Graph of the derivative of f

(a) Determine *all* values of *x* for which:

(i) *f* has critical point(s) ____________________

(ii) *f* has local maximum(s) ____________________

(iii) *f* has local minimum(s) ____________________

(iv) *f* has inflection point(s) ____________________

(b) Give one interval over which *f* is concave down. ____________________

(c) Give the largest interval over which *f* is increasing. ____________________

(6.) (8 pts) [Show all work.]

If y satisfies the equation

$$y^2 + 2xy - 3x = 0,$$

(a) find $\frac{dy}{dx}$.

(b) Based on your answer to part (a), is the graph increasing, decreasing, or neither (*ie.*, tangent horizontal or undefined) at the point (1,1)? Explain.

(7.) (12 pts) A laboratory study investigating the relationship between diet and weight in adult humans found that the weight, *W*, of a subject, in pounds, was a function, *f*, of the daily average number of calories, *c*, consumed by the subject. In terms of diet and weight, interpret the following statements or expressions. [Be certain to include units and write in sentences.]

(a) $f(1800) = 155$

(b) $f'(2000) = 0$

(c) $f^{-1}(162)$

(d) What are the units of $f'(c)$?

(8.) (12 pts) From Exam I, we have that the population of Michigan can be approximated by

$$\boldsymbol{P} = f(t) = 7.8(1.0058)^t,$$

where t is the number of years since the beginning of 1960 and P is in millions.

(a) Determine the average rate of change in the population of Michigan between 1960 and 1980. [Be certain to include units and express your answer as a complete sentence.]

(b) Determine the (instantaneous) rate of change of the population of Michigan at the beginning of 1980. [Again, use units and a sentence. Show your work.]

(c) Which is greater—the average rate of change between 1960 and 1980 or the instantaneous change in 1980? Use a graph or tables to give a convincing argument that the rate that you *found* to be greater should indeed *be* greater.

(d) Is there some time, t, such that the instantaneous rate of change of P is equal to the average rate of change from 1960 to 1980? If so, approximate t. If not, explain why not.

(9.) (10 pts) Determine *a*, *b*, and *c* so that the graph of the function $f(x) = x^3 + ax^2 + bx + c$ has
a local maximum at $x = -2$,
a local minimum at $x = 1$,
and passes through the point (0,2). [Show your work.]