MATH 115 — MIDTERM EXAM # 2

Department of Mathematics
University of Michigan

March 20, 2002

NAME: _____ ID NUMBER: _____

SIGNATURE: _____

INSTRUCTOR: _____ SECTION NO: _____

General Instructions: Do not open this exam until you are told to begin. This test consists of 12 questions on 10 pages (including this cover sheet). The last page is blank and is for your use as a worksheet. The exam is worth 100 points. Do not separate the pages of exam. If any pages do become detached, write your name on them and point them out to your instructor when you turn in the exam.

Please read the instructions for each individual exercise carefully. Show an appropriate amount of work for each exercise so that graders can see not only the answer but also how you obtained it. If you use graphs or tables to obtain an answer, be certain to provide an explanation (and a sketch of the graph, if that is the method) to make it clear how you arrived at your solution. Use units where appropriate.

You are allowed two sides of a 3 by 5 card of notes and are expected to use your calculator.

PROBLEM	POINTS	SCORE
1	12	
2	7	
3	7	
4	8	
5	9	
6	10	
7	10	
8	8	
9	4	
10	7	
11	9	
12	9	
TOTAL	100	

1. (2 points each) **True or False**. Circle True only if the statement is always true.

(a) If f' is increasing, then f is increasing. T F

(b) If f is an exponential function, then $\frac{d}{dx}\ln f(x)$ is constant. T F

(c) If $f''(x) = 0$ for all x, then f is a constant function. T F

(d) There is a function f so that $f(x) > 0$, $f'(x) < 0$, and $f''(x) < 0$ for all x. T F

(e) If $f''(x) < 0$ for all x, then $f(x) \leq f(0) + f'(0)x$ T F

(f) If $f'(x) = 0$, then f has either a relative maximum or relative minimum at x. T F

2. (7 points) The function g has a continuous derivative whose values are given in the following table. There is no more than one critical point of g between any two consecutive x-values in the table.

Note that the table gives values of $g'(x)$, **NOT** $g(x)$.

x	0	1	2	3	4	5	6	7	8	9	10
$g'(x)$	-9	-2	2	1	-1	-3	-6	-5	-4	2	10

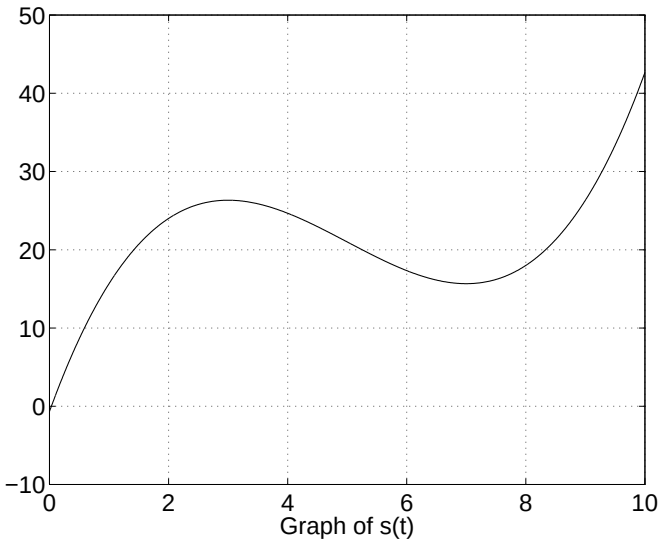
(a) Estimate the x-coordinates of the critical points of g for $0 < x < 10$.

(b) For each critical point found in part (a), determine if it corresponds to a local maximum or minimum of the function g. Be sure to explain.

3. (7 points) Suppose that $f(T)$ is the cost to heat my house, in dollars per day, when the outside temperature is T degrees Fahrenheit.
(**a**) What does $f'(23) = -0.17$ mean in the context of this problem?

(**b**) If $f(23) = 7.54$, and $f'(23) = -0.17$, what is the approximate cost to heat my house when the outside temperature is 20 degrees Fahrenheit?

4. (8 points) An object is moving on a straight line so that its distance (measured in feet) to the right of a fixed point on the line at time t (measured in seconds) is given by the function s whose graph is in the following figure.



(a) At what times (approximately) is the object moving to the right? to the left?

(b) At what times (approximately) does the object have positive acceleration? negative acceleration? (Explain what properties of the graph give you this information.)

(c) At what times (approximately) is the velocity of the object increasing? Explain.

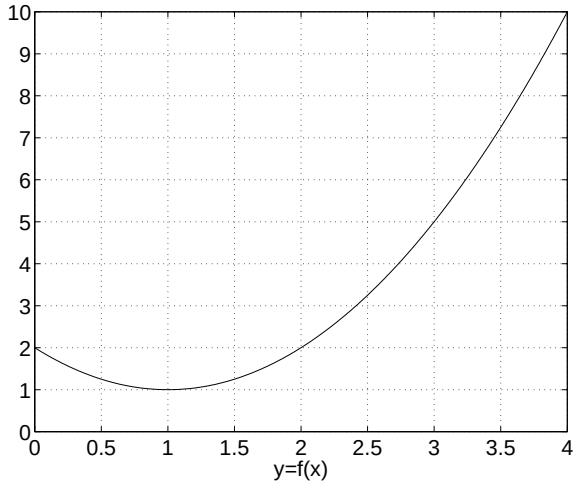
5. (9 points) Find the equation of the tangent line to the curve $2x^2y^2 - x^3 - y^5 + 1 = 0$ at the point $(2, 1)$.

6. (10 points) (**a**) Find the Taylor polynomial of degree two that approximates the function $(1 + 2x)^{\frac{3}{2}}$ at $x = 0$ (Show your work!).

(**b**) What is the local linearization of $(1 + 2x)^{\frac{3}{2}}$ near $x = 0$?

(**c**) Is the local linearization of $(1+2x)^{\frac{3}{2}}$ an overestimate or underestimate of the function? Why?

7. (10 points) Let f and g be functions with the following graphs.



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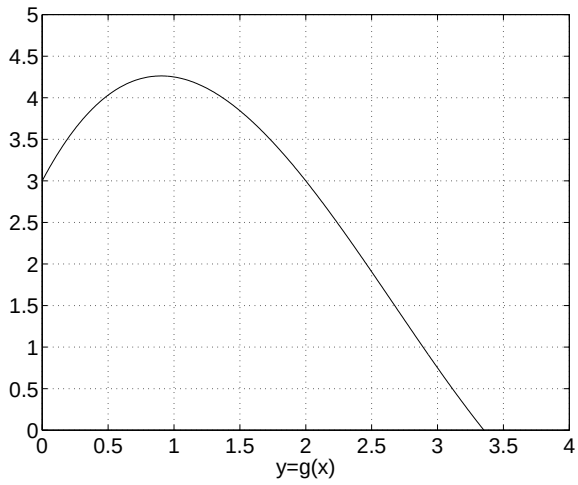


Use the graphs to estimate each of the following derivatives. Show your work and circle your answers.

(a) $h'(2)$ if $h(x) = f(x)g(x)$

(b) $h'(2)$ if $h(x) = \frac{f(x)}{g(x)}$

(c) $h'(2)$ if $h(x) = f(g(x))$

8. (8 points) On the set of axes provided, draw the graph of a smooth function f such that this function has all of the following properties.

(a) $f(3) = 2$

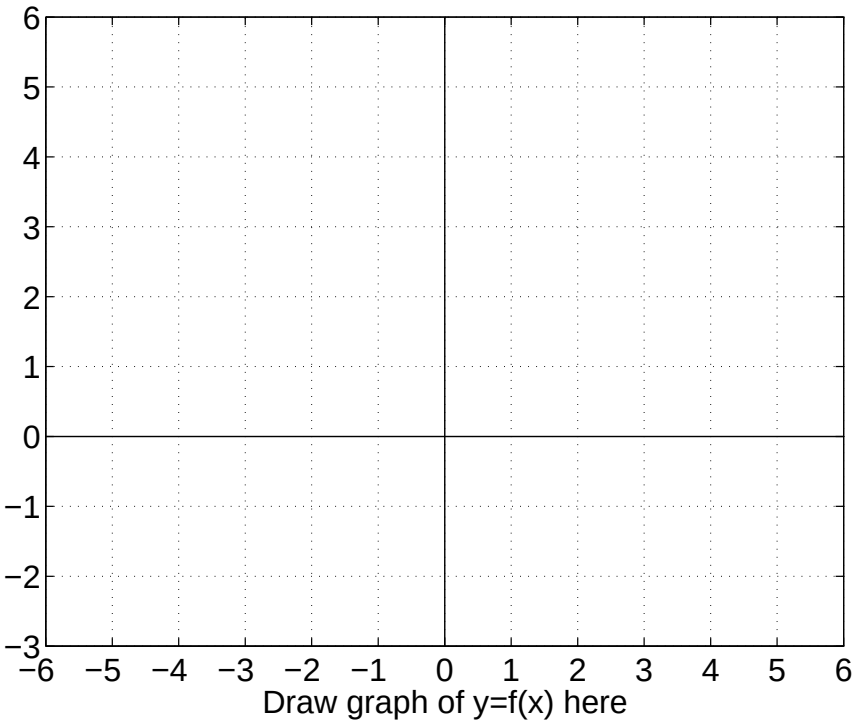
(b) $f' < 0$ for $x < 0$

(c) $f' > 0$ for $x > 0$

(d) $f'' > 0$ for $x < 3$

(e) $f'' < 0$ for $x > 3$

(f) the graph of f does not pass through the origin



(b) Is it possible that $f(x) = 0$ for some $x > 3$? Explain.

9. (4 points) Let f, g be functions such that $f''(x) > 0$ and $g''(x) < 0$ for all x. In how many points can the graphs of f and g intersect? Circle all possible answers.

(i) no points

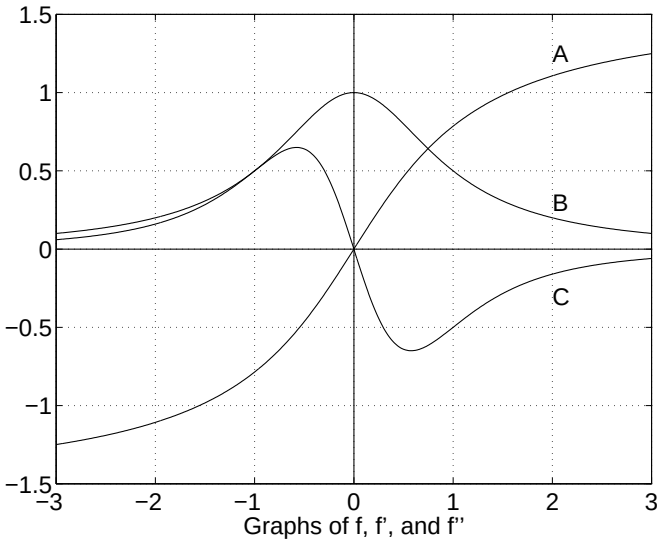
(ii) 1 point

(iii) 2 points

(iv) 3 points

(v) infinitely many points

10. (7 points) (a) The figure below shows graphs of a function f and its first and second derivatives, f' and f''. Identify by the label on the graph which function is f, which is f', and which is f''.



A is the graph of ____

B is the graph of ____

C is the graph of ____

(b) Give a clear explanation of your reasoning for the choices you made in part (a).

11. (9 points) Recall Hanky- and Pankytowns? On the first exam, we saw that the population of Pankytown, in thousands, could be modeled by

$$P(t) = 50(0.8)^t$$

where t is the number of months after February, 2001 when valentines were banned in Pankytown.

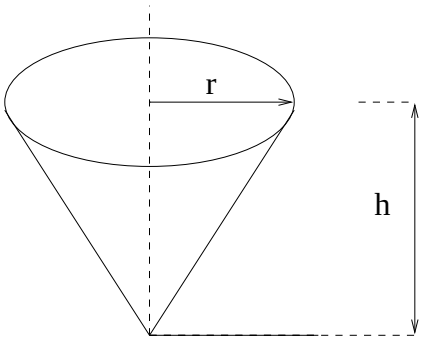
(a) At what rate was the population of Pankytown changing in May of 2001?

(b) We also found in Exam 1 that the population of Hankytown (in thousands) was given by

$$H(t) = 9\cos\left(\frac{\pi t}{6}\right) + 11$$

with $t = 0$ representing the month of February, 2001. Is there a time (or times) during the first 18 months after February, 2001, that the models indicate that the populations of Pankytown and Hankytown are changing at the same rate? If so, when? If not, explain why not. Clearly explain how you found your answer.

12. (9 points) Fluid flows out of the bottom of a cone-shaped vessel at the rate of 3 cubic cm per second (see figure below). If the radius of the cone is one-third of its height, how fast is the height of the fluid changing when the fluid is 6 cm deep in the center of the cone. Be sure to show your work and give the correct units in your answer. (Remember that the volume of a cone is $\frac{1}{3}\pi r^2 h$.)



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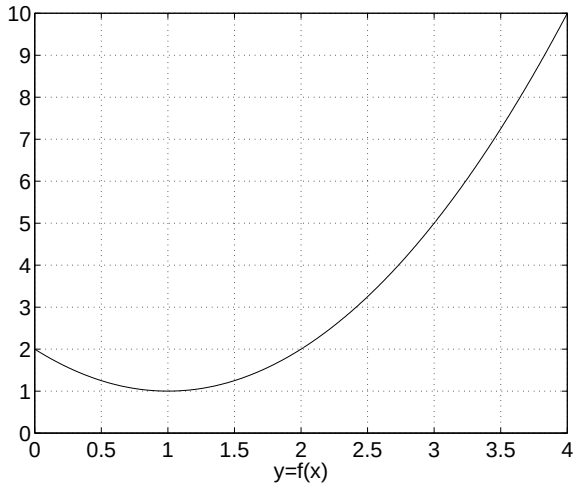
CORRECTED GRAPHS FOR PROBLEM 7 ON PAGE 5 OF THE EXAM

Instructions:

1. Use these graphs to answer problem 7. Do not use the graphs in the exam packet. Major credit deductions will be taken for using the graphs in the exam.

2. Show your work for problem 7 in the exam packet. DO NOT show your work on this page. DO NOT turn this page in. Any of these colored pages turned in with exams will be discarded immediately.



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