MATH 115 — FINAL EXAM

DEPARTMENT OF MATHEMATICS
University of Michigan

April 23, 2002

NAME: ____________________________ ID NUMBER: ___________________________

SIGNATURE: ____

INSTRUCTOR: ____________________ SECTION NO: ___________________________

General Instructions: Do not open this exam until you are told to begin. This test consists of 11 questions on 12 pages (including this cover sheet). The last page is blank and is for your use as a worksheet. The exam is worth 100 points. Do not separate the pages of exam. If any pages do become detached, write your name on them and point them out to your instructor when you turn in the exam.

Please read the instructions for each individual exercise carefully. Show an appropriate amount of work for each exercise so that graders can see not only the answer but also how you obtained it. If you use graphs or tables to obtain an answer, be certain to provide an explanation (and a sketch of the graph, if that is the method) to make it clear how you arrived at your solution. Use units where appropriate.

You are allowed two sides of a 3 by 5 card of notes and are expected to use your calculator.

PROBLEM	POINTS	SCORE
1	14	
2	7	
3	9	
4	9	
5	10	
6	10	
7	12	
8	8	
9	4	
10	5	
11	12	
TOTAL	100	

1. (14 points) **(a)** An antiderivative of

$$f(t) = \sin(t) + \frac{1}{\cos^2 t}$$

is $F(t) =$ ______________________________.

(b) If $\int_0^3 f(x)\,dx = 3$, then $\int_0^3 (f(x) + 2)\,dx =$ ______________.

(c) A cubic polynomial (3rd degree polynomial) always has a

(i) local maximum	**Yes**	**No**
(ii) local minimum	**Yes**	**No**
(iii) global maximum	**Yes**	**No**
(iv) global minimum	**Yes**	**No**
(v) inflection point	**Yes**	**No**

(d) The exact value of c such that

$$\int_0^c x\sqrt{x}\,dx = \frac{4}{5}$$

is $c =$ ____________________.

2. (7 points) What is the largest area a rectangle can have if its base lies on the x-axis and its upper vertices lie on the curve $y = a^2 - x^2$? (Your answer will be in terms of a. Show your work.).

3. (9 points) The water level in an an underground tank varies periodically every 7 hours, oscillating between a maximum level of 4.6 feet and a minimum of 3.2 feet.

(a) If the water reaches a maximum height at 9am on a certain day, write a formula using the sine or cosine function, for the height h as a function of time t, where t is measured in hours past midnight of that day.

(b) What are the period and amplitude of your function? (Use units, if appropriate.)

Period ____________________________ Amplitude ____________________________

(c) At what rate is the water rising or falling (indicate which) at 2pm on that day? (Be sure to use units in your answer.)

4. (9 points) Last week David ran the Naked Mile, starting out at a fast pace with the idea of winning the race. His friend, John rode along on his bike to clock David's times. However, at the end of the race the police were chasing David so that he kept on running to avoid being arrested. John followed and recorded David's speeds at 5 minute intervals. David was slowing down all the time, but fortunately for him, the policemen were unable to catch him. They finally gave up chasing him after 25 minutes. David continued for an additional five minutes before stopping.

The speeds John clocked are recorded in the following table. In recounting his experience, David wondered how far he actually ran in the half hour. Help him out by answering the questions in parts (a) and (b). (Be sure to show your work when answering the questions).

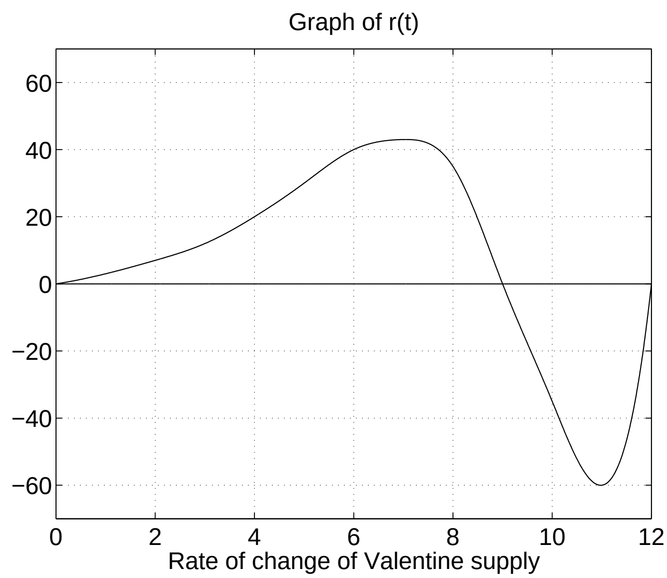
time (in minutes):	0	5	10	15	20	25	30
speed (in miles per minute):	.2	.16	.14	.12	.12	.1	.05

(a) Assuming that David's speed never increases throughout the run, use the data in the table to determine the best estimate for the total distance that David ran during the 30 minutes.

(b) To be sure of estimating the distance David traveled to within .15 miles, how frequently would John have needed to record the measurements of David's speed?

5. (10 points) Consider the family of functions defined for $x \geq 0$ by $y = axe^{-bx}$ where a and b are positive numbers. Find all members of this family that pass through the point $(2, 3)$ and have a critical point at $x = 2$. Determine if this critical point is a local maximum, local minimum, or neither. If it is a local maximum or minimum, is it also a global maximum or minimum on this domain (i.e. for $x \geq 0$)?

6. (10 points) Recall that Hankytown, the community famed for making valentines, has a fluctuating population based on the influx of migrant valentine makers. The number of valentines in the city coffers varies also according to the season of the year. The graph below shows the rate, $r(t)$ (in 1000's of valentines per month), at which the supply of valentines changes over a 12 month period, where $t = 0$ corresponds to the beginning of January.



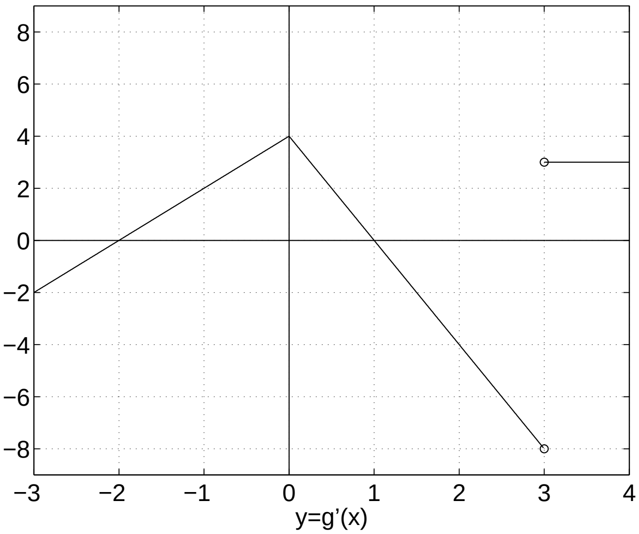
(a) Over what period of time did the valentine supply grow?

(b) When was the supply of valentines growing most rapidly?

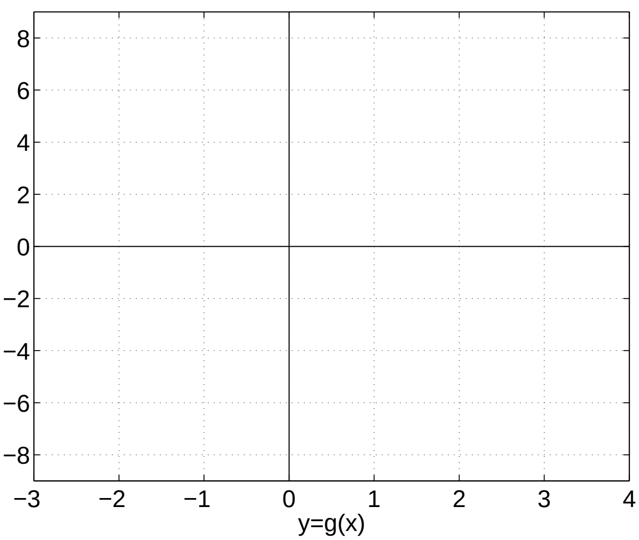
(c) Write a mathematical expression giving the average rate at which the valentine supply is changing over the first four months shown in the graph.

(d) Given that there were 25,000 valentines in the warehouse at the beginning of the period shown, write a mathematical expression for the total number of valentines in the warehouse at the end of the 12 month period. Were there more or fewer than 25,000 valentines at this time. How do you know?

7. (12 points) A function g is known to be continuous and the graph of its derivative, g', for $-3 \leq x \leq 4$ is given in the following figure.



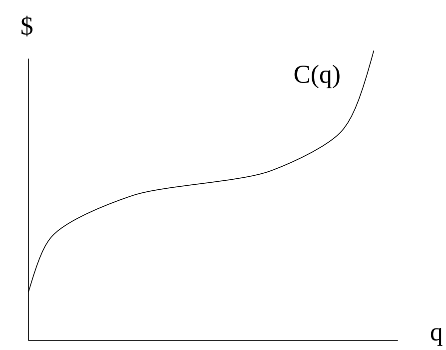
(a) Given that $g(-3) = 0$, sketch the graph of g on the axes provided below. In the space below the figure, give the coordinates of ALL *critical points* of g.



(b) Critical Points:

8. (8 points) (a) It is a fact from economics that the average cost $C(q)/q$ of producing $q > 0$ units of a quantity is minimized when this average cost is equal to the marginal cost. Show analytically why this is so.

(b) Using the graph of $C(q)$ shown below, a typical cost function as in the text, indicate on the q-axis the value of q_0 which minimizes the average cost. Explain graphically why the average cost is equal to the marginal cost at this point.



9. (4 points) Explain either why the following statement is always true or show a function for which it is false.

"If f is a differentiable function defined for all x and if f has a local maximum at x_0 and a local minimum at x_1, then $f(x_0) \geq f(x_1)$."

10. (5 pts) Upon returning home this summer, you meet a good friend who is just now graduating from high school. He has done very well in precalculus and has a good understanding of the graphs, tables, and formulas that a good student in precalulus should know. He is planning to come to U-M next year and will take Math 115. He has heard that one of the basic concepts in 115 is something called a derivative.

In the space below, explain what you would say to give your friend a good idea of what the derivative means, illustrating this in as many ways as you believe will help your friend understand the concept.

11. (12 points) Ms. Manufacturer, a producer of diamond-studded widgets, finds that her company can sell q widgets per week if they are priced at p each, where

$$q = 100 - 2p$$

and p is in hundreds of dollars. Her cost, also measured in hundreds of dollars, for producing q widgets is

$$C(q) = 100 + 10q + \frac{1}{2}q^2.$$

(a) How many widgets should her company manufacture each week to achieve the least cost per widget? That is, the least average cost. (Be sure to show your work.)

(b) Determine the formula for the revenue $R(q)$ received each week if q widgets are sold.

(c) How many widgets should Ms. Manufacturer's company produce each week in order to maximize profits? At what price should the widgets be sold?

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Solution to graph for problem 7.