Solution Guide

MATH 115 — MIDTERM EXAM # 1

DEPARTMENT OF MATHEMATICS
University of Michigan

February 6, 2002

NAME: Key ______________ ID NUMBER: ______________

SIGNATURE: ______________________

INSTRUCTOR: ______________ SECTION NO: ______________

General Instructions: Do not open this exam until you are told to begin. This test consists of 12 questions on 9 pages (including this cover sheet). The last page is blank and is for your use as a worksheet. The exam is worth 100 points. Do not separate the exam pages. If any pages do become detached, write your name on them and point them out to your instructor when you turn in the exam.

Please read the instructions for each individual exercise carefully. Show an appropriate amount of work for each exercise so that graders can see not only the answer but also how you obtained it. If you use graphs or tables to obtain an answer, be certain to provide an explanation (and a sketch of the graph, if that is the method) to make it clear how you arrived at your solution. Use units where appropriate.

You are allowed two sides of a 3 by 5 card of notes but no other books or manuals.

PROBLEM	POINTS	SCORE
1	12	
2	6	
3	6	
4	10	
5	10	
6	5	
7	5	
8	9	
9	9	
10	8	
11	8	
12	12	
TOTAL	100	

1. (2 points each) **True or False**. Circle True only if the statement is always true.

(a) The inverse function of $g(t) = (1.04)^t$ is $g^{-1}(t) = \frac{1}{(1.04)^t}$. T F

(b) $\ln(2^x + 2^{-x}) = 0$ T F

(c) If $22 = 18e^{2k}$, then $k = 1.003$. T F

(d) $\log(67.34(1.03)^t) = t(\log(67.34) + \log(1.03))$ T F

(e) The graph of the function $s(t) = 2\sin(2t + 3)$ is the graph of the function $y = 2\sin(2t)$ shifted 3 units to the left. T F

(f) If f' is increasing, then f is increasing. T F

2. (6 points) A function $f(x)$ has values given in the following table. Estimate the value of its derivative at $x = 1$.

x	.9	.98	.996	1.0	1.004	1.02	1.1
$f(x)$	.7969	.8342	.8410	.8427	.8444	.8508	.8802

(any of these)

$$f'(1) \approx \frac{f(1.004) - f(1)}{.004} = \frac{.8444 - .8427}{.004} = .425$$

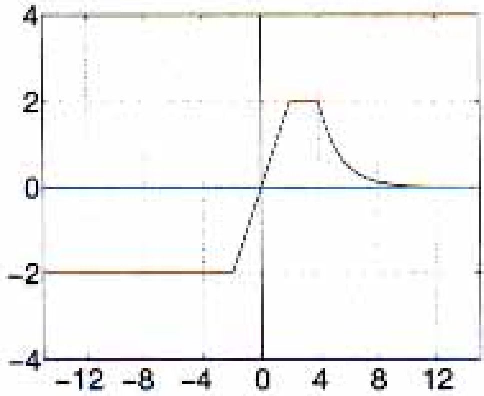
or

$$f'(1) \approx \frac{f(.996) - f(1)}{-.004} = \frac{.8410 - .8427}{-.004} = .425$$

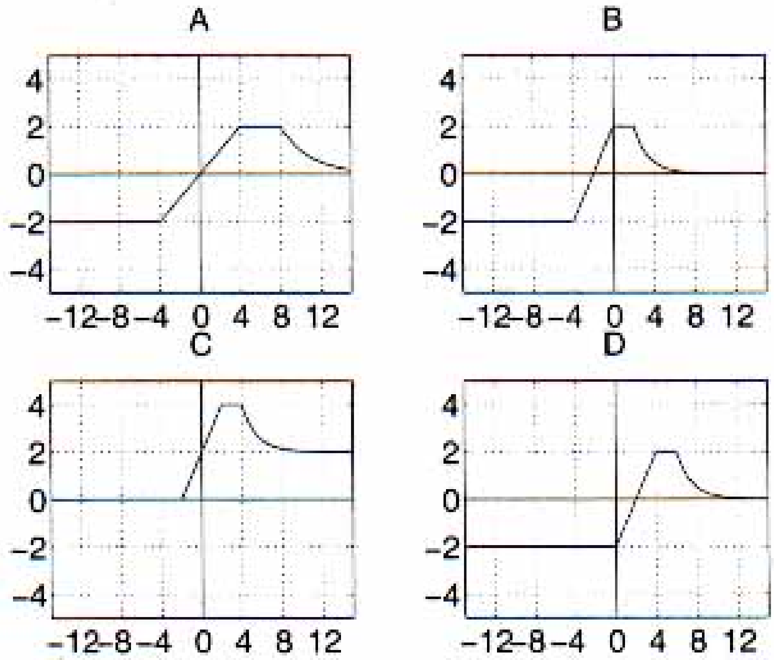
or

$$f'(1) \approx \frac{f(1.004) - f(.996)}{.008} = \frac{.8444 - .8410}{.008} = .425$$

3. (6 points) A function $f(x)$ has the following graph.

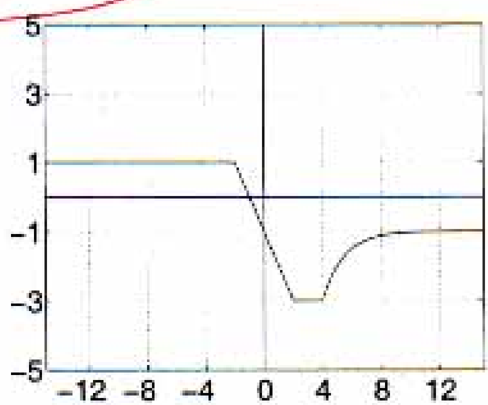


a. Of the four graphs given below, which is the graph of $f(x+2)$? B



b. Write a formula in terms of f for the function whose graph is given in the figure below.

$y = -f(x) - 1$



4. (10 points) During the last century, home prices in Ann Arbor have changed a great deal. Let $N(t)$ be the median price (in thousands of dollars) of new houses, and $P(t)$ be the median price of previously owned houses on the market in Ann Arbor t years after the year 1900 AD. Explain the practical meaning (i.e., in terms of housing and money) of each of the following equations by rewriting as an English sentence (without the symbols $N(t)$ and $P(t)$).

(a) $N(100) = N(80) + 101$

The median price of new homes in Ann Arbor in the year 2000 was \$101,000 more than the median AA new home prices in 1980.

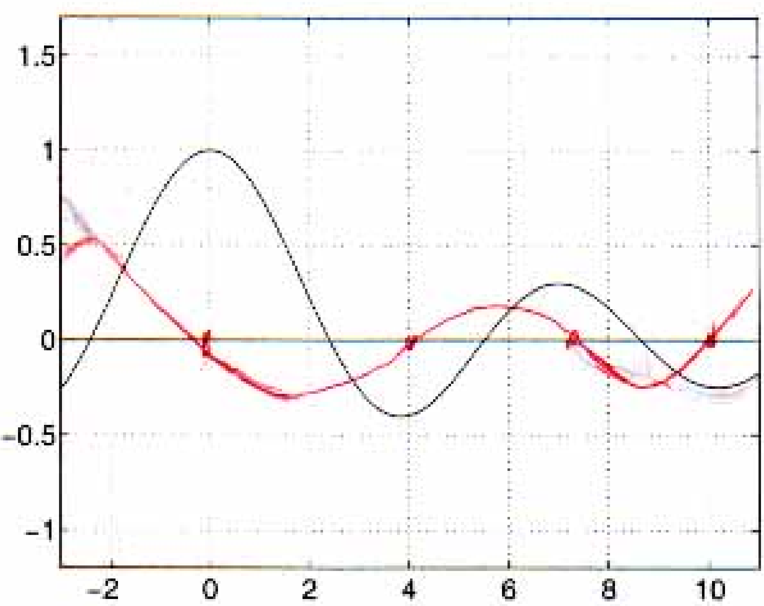
(b) $N(100) = 1.25P(100)$

In the year 2000, the median price for new AA homes was 1.25 times (or 25% greater than) the median for previously-owned homes.

(c) $P^{-1}(100) = 80$

In 1980 the median AA new home price was \$100,000.

5. (10 points) The graph of $y = f(x)$ is shown in the figure below. On the same set of axes, sketch a graph of the derivative of f.



6. (5 points) There were 238 million bushels of wheat grown in Michigan in 1990 and the wheat sold for \$2.21 per bushel that year. In 1992, there were 242 million bushels grown, and wheat sold for \$2.00 per bushel in 1992.

(a) What was the revenue from wheat in Michigan in 1990?

Rev in 1990 = 238(2.21) = \$525.98 million

In 1992?

Rev in 1992 = 242(2.00) = \$484 million

(b) What was the average rate of change of the revenue from wheat over the period of time from 1990 to 1992?

Avg Rate of Chg of Rev. from 1990-1992 $= \dfrac{484-525.98}{2} = -20.99$ million dollars/year

7. (5 points) A function f is known to have positive average rate of change on the interval from $x = 2$ to $x = 4$. Which of the following numbers are possible values for $f'(3)$? Circle all that apply.

(i) 3

(ii) 0

(iii) -2

all are possible...

8. (9 points) Values of functions f and g are given in the following table. The function g is known to be invertible. Determine:

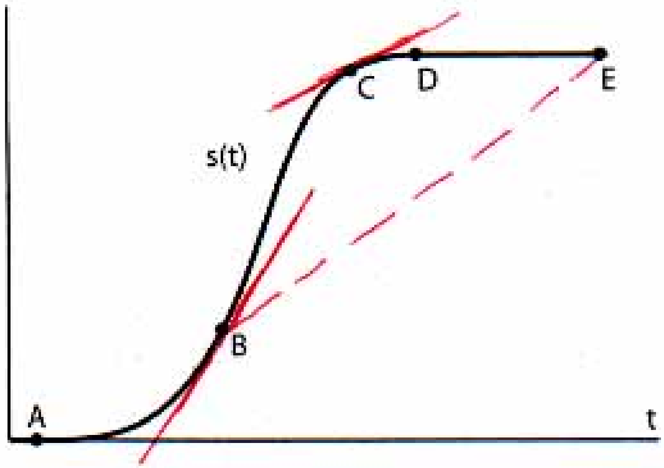
(a) $f(g(3))$ $= f(5) = 2$

(b) $f(f(3))$ $= f(4) = 6$

(c) $f(g^{-1}(3))$ $= f(2) = 7$

x	1	2	3	4	5
$f(x)$	0	7	4	6	2
$g(x)$	1	3	5	7	9

9. (9 points) The following graph shows the distance $s(t)$ travelled along a highway by a car driving away from a city. From the shape of the graph, explain answers to the questions.



(a) Was the car going faster at the time corresponding to point B or the time corresponding to point C? Explain.

The car was going faster at the time corresponding to point B because the slope of the tangent to the curve is more positive at point B than at point C. Since the curve represents distance, the slope gives the rate of change of distance, or velocity.

(b) Was the average velocity of the car for the time between points B and E greater than or less than the car's instantaneous velocity at the time corresponding to the point B? Explain.

The average rate of change (or average velocity) between points B + E is given by the slope of the dashed line between B + E. The slope of this line is less than the line tangent to the curve at B. Thus, the car's velocity at the time indicated by point B is greater.

(c) What was the car doing during the times corresponding to points between D and E? Explain.

Between points D + E on the graph, the function is constant and the slope of the curve between these points is zero. Thus, for the times corresponding to the points between D + E, the car was stopped.

10. (8 points) Let $f(x) = \ln(\sin x)$. Use your calculator and the **limit definition** of the derivative to approximate the instantaneous rate of change of f at x = 1. In order to receive full credit, you must show your work and indicate the values that you use to come up with your approximation. (Note: be sure that your calculator is set to radian mode.)

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

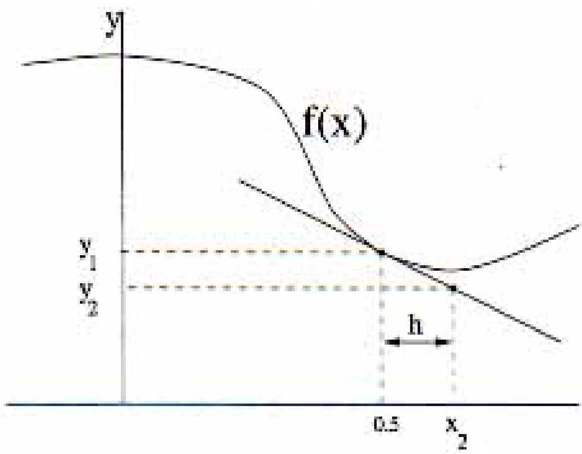
$$= \lim_{h \to 0} \frac{\ln(\sin(1+h)) - \ln(\sin 1)}{h} \quad (*)$$

taking small values of h, we have

h	$(*)$
.01	.63506
.001	.64139
.0001	.64202
−.0001	.64202
−.001	⋮
−.01	⋮

$\rightarrow f'(1) \approx .642$

11. (8 points) In the figure below, it is given that $f(0.5) = 3$, $f'(0.5) = -2$, and $h = 0.1$. Determine the values of y_1, y_2, and x_2.



(or can find equa of tangent to get x_2 & y_2 ...)

Given: $f(0.5) = 3 \rightarrow y_1 = 3$

Given $f'(0.5) = -2$ & $h = 0.1$

$\rightarrow \frac{\Delta y}{\Delta x} = \frac{\Delta y}{.1} = -2$

$\Delta y = -.2$

$x_2 = 0.5 + 0.1 = 0.6$

$y_2 = y_1 - .2 = 3 - .2 = 2.8$

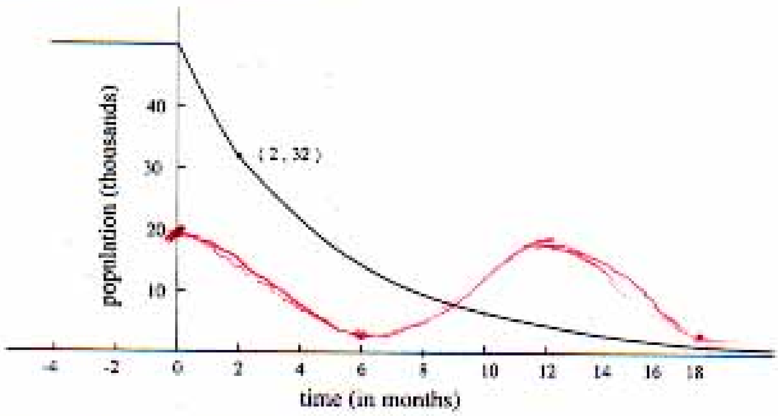
$y_1 =$ 3

$y_2 =$ 2.8

$x_2 =$ 0.6

12. (12 points) For years the town of Hankytown has struggled with the city budget. The economy in Hankytown is based solely on the sale of valentines. Thus, the population in Hankytown varies each year from a high of 20,000 people (mostly migrant valentine makers) in February, to a low of 2,000 people in August.

On the other hand, Pankytown, just down the road from Hankytown, had a very stable economy. The population of 50,000 people had held steady for some time. However, in February of last year, the mayor of Pankytown decided to ban the purchase or sale of valentines. The citizens were outraged–rightfully so! People began to move as far away from Pankytown as they could get. The graph of $P(t)$ in the figure below shows what has happened to the population of Pankytown in the months since the ban took place.



(a) Determine a formula for an exponential function giving the population, P (in thousands), of Pankytown as a function of t, with $t = 0$ representing the first of February last year. [Note: the formula will only model the population for $t \geq 0$ in this picture.]

$P(t) = 50(b)^t$ and $P(2) = 50(b)^2 = 32$

so $b^2 = \frac{32}{50} = .64$

$b = .8$

$P(t) =$ $50(0.8)^t$

(b) (i) On the same set of axes with $P(t)$ above, sketch a graph of the function which represents the population of Hankytown, H (in thousands), as a function of t, with $t = 0$ representing the first of February.

(ii) Determine a formula for a trigonometric function giving the population $H(t)$.

ampl = 9

period = 12 $\Rightarrow B = \frac{2\pi}{12} = \frac{\pi}{6}$

upward shift (D) = 11

$H(t) =$ $9\cos\left(\frac{\pi}{6}t\right) + 11$

(c) Is there a time (or times) that these models indicate that the populations of Pankytown and Hankytown are the same? If so, when? If not, explain why not.

Using a graphing calculator, we see that the curves intersect at $t \approx 8.3$ months. Thus in approx Oct of last year the populations were the same. From then on (if the trend continues) Pankytown will have fewer people than Hankytown.