MATH 115 — FINAL EXAM

Department of Mathematics
University of Michigan

April 21, 2003

NAME: ____

INSTRUCTOR: ____________________ SECTION NO: ______________

1. Do not open this exam until you are told to begin.
2. This exam has 10 pages including this cover. There are 11 questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Pay particular attention to the instructions for Problem 1, but read *all* instructions for each individual exercise carefully.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it.
6. If you use graphs or tables to obtain an answer, be certain to provide an explanation and a sketch of the graph to make it clear how you arrived at your solution.
7. Include units in your answers where appropriate.
8. You are allowed two sides of a 3 by 5 card of notes. You may also use your calculator.
9. Please turn off all cell phones.

PROBLEM	POINTS	SCORE
1	16	
2	5	
3	6	
4	12	
5	9	
6	8	
7	8	
8	9	
9	9	
10	9	
11	9	
TOTAL	100	

1. True or **False**–no explanation necessary. Circle **True** only if the statement is *always* true. You are encouraged to answer these problems only if you are sure of your answer.

Scoring will be:

- 2 points for each correct answer,
- 0 points for not answering, and
- -1 point for each incorrect answer.

Assume that all functions are continuous and differentiable.

(a) If f is increasing, then f' is increasing. **True** **False**

(b) If $y = \pi^5$, then $y' = 5\pi^4$. **True** **False**

(c) If f' is increasing, then the graph of f lies above the graph of any line that is tangent to the curve. **True** **False**

(d) If $f''(a) = 0$, then f has an inflection point at $x = a$. **True** **False**

(e) If f'' is negative at a critical point, then f has a local maximum at that point. **True** **False**

(f) If $a \neq b$, then $\int_a^b f(x)\,dx \neq 0$. **True** **False**

(g) If $f(x) \leq g(x)$ for all x on the interval $[2, 6]$, then $\int_2^6 [g(x) - f(x)]\,dx \geq 0$. **True** **False**

(h) If $\int_a^b (2f(x) + g(x))\,dx = 5$ and $\int_a^b g(x)\,dx = 2$, then $\int_a^b f(x) = 3$. **True** **False**

2. (5 points) The temperature, A, measured in degrees Fahrenheit, of the water near the surface of a small lake t days after the beginning of fall is described by $A = f(t)$.

Explain the meaning of the statement "$f'(30) = -2$".

3. (6 points) A continuous, differentiable function f is defined for $x \geq 0$, and satisfies

- f has exactly one critical point,
- $f(0) = 0$ and $f(3) = 2$,
- $f'(1) = 0$, and
- $\lim_{x \to \infty} f(x) = 0$.

Circle each of the following conditions that are possible.

f has a local maximum at $x = 1$.

f has a local minimum at $x = 1$.

f has neither a local maximum or a local minimum at $x = 1$.

f has a global maximum at $x = 1$.

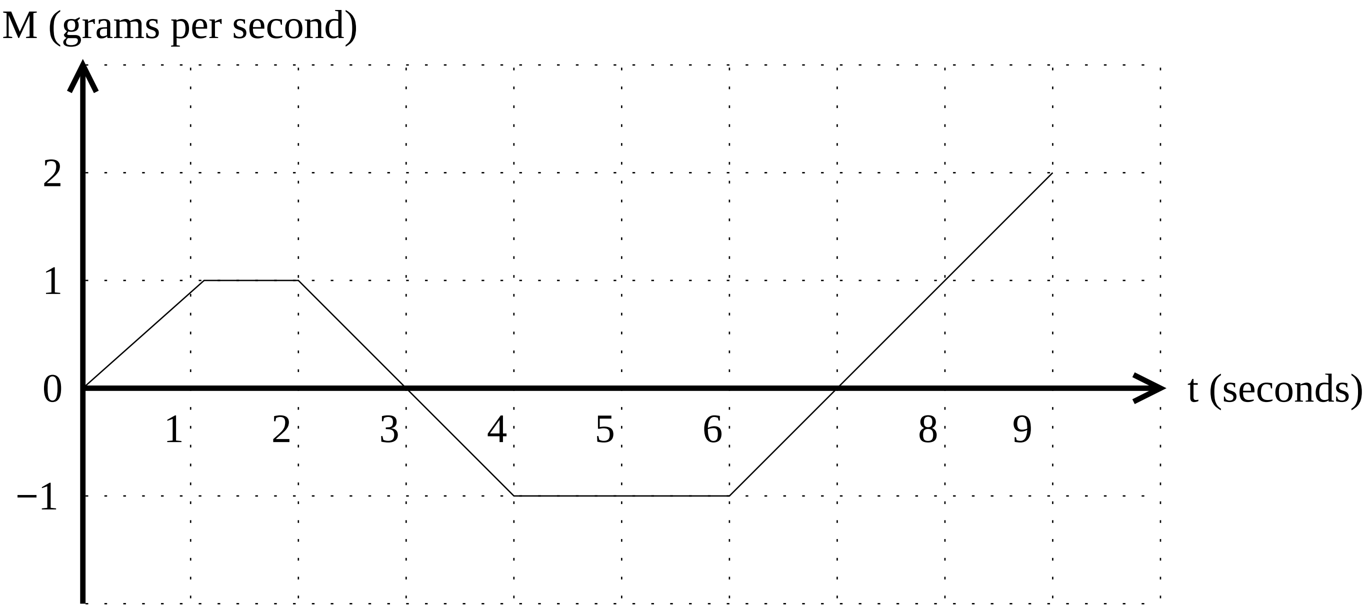
f has a global minimum at $x = 1$.

4. (12 points) **(a)** Give the limit definition of the derivative of a function f at a point a.

(b) Use the limit definition of the derivative to find $g'(x)$ for the function $g(x) = 2x^2 - 3x$. [Be sure to show all of your work!]

(c) Use the Fundamental Theorem of Calculus to find $\int_2^4 (4x - 3)\, dx$. [Note: You must show your work to receive credit.]

5. (9 points) A substance, B, is one of several substances involved in a complex chemical reaction. At certain times during this reaction, substance B is produced by the reaction while at other times it plays the role of a reactant and is consumed. Given that enough reactants are present, the rate M, of production of substance B is approximated by the function whose graph is given below.

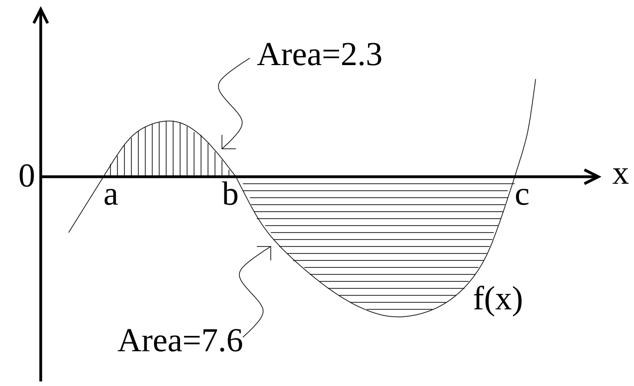


(a) Over what interval(s) is the amount of substance B increasing?

(b) At what time during the reaction is the least amount of substance B present? Explain.

(c) The reaction takes 9 seconds to complete and will not proceed if there is no substance B present. There is a value, V, such that if the reaction begins with V or fewer grams of substance B, then the reaction will not proceed to completion. Find the value of V, and explain your answer.

6. (8 points) Use the figure below to calculate the numerical values of the definite integrals in parts (a) through (d). You need not show your reasoning.



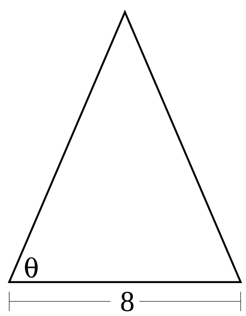
(a) $\displaystyle\int_a^b f(x)\,dx =$ ____________

(b) $\displaystyle\int_b^c f(x)\,dx =$ ____________

(c) $\displaystyle\int_a^c f(x)\,dx =$ ____________

(d) $\displaystyle\int_b^a f(x)\,dx =$ ____________

7. (8 points) An isosceles triangle has a base of length 8 meters. If θ denotes the angle opposite one of the two equal sides, and if θ is increasing at a constant rate of of 0.1 radians per second, how fast is the area of the triangle increasing when $\theta = \pi/6$?



8. (9 points) The table gives the values of a function obtained from an experiment.

x	0	1	2	3	4	5	6	7	8
$f(x)$	9.3	9.1	8.3	6.9	3.3	-.6	-1.7	-3.5	-6.7

(a) Using these values, estimate $\int_0^8 f(x)\,dx$ using 4 subintervals and right hand endpoints.

(b) If f is known to be a decreasing function, can you determine if your answer in part (a) is an over or underestimate? If so, which is it? If not, why not. Be sure to explain your answer.

9. (9 points) An ecologist is studying the biodiversity of an environment near the top edge of a windswept cliff. One statistic of interest to her is the distribution of biomass throughout the environment. If x measures the horizontal distance from the cliff edge in meters, there is only one species of tussock grass that grows for $1 \leq x \leq 20$. Along the first meter from the cliff's edge, nothing grows; and beyond 20 meters from the cliff, various other plant species thrive.

A typical tussock grass plant located x meters from the cliff edge has mass $\frac{2}{9}x^3 + \frac{3}{2}x^2 + 3$ kilograms per plant, and there will be $\frac{1}{2x^2}$ such plants per square meter.

(a) For $1 \leq x \leq 20$, find the distance from the cliff which minimizes the biomass per square meter. Show your work.

At a distance of ________________ from the cliff's edge the biomass per square meter is minimized.

(b) What is the maximal biomass per square meter in this region? Explain.

The maximal biomass per square meter in this region is ________________.

10. (9 points) A piece of wire of length 40 cm is cut into two pieces. One piece is made into a circle; the rest is made into a square.

Find the lengths of each piece of wire so that the sum of the areas of the circle and square is a minimum. [Be sure to show *all* of your work and clearly identify your answers.]

11. (9 points) Let $s(t)$ give the position of an object along a straight line at time t and let $v(t)$ denote its instantaneous velocity at time t.

(a) Give the definition of the *average velocity* of the object over the time interval from $t = a$ to $t = b$.

(b) Give the definition of the *average of the velocity function* over the interval from $t = a$ to $t = b$.

(c) Is the *average velocity* of the object over the time interval from $t = a$ to $t = b$ equal to the *average of the velocity function* over this time interval? If so, explain why. If not, explain why not.

Please rewrite your name and section number.

NAME: ____

SECTION NO: ____________