MATH 115 — FIRST MIDTERM EXAM

February 10, 2004

NAME: _____

INSTRUCTOR: _____ SECTION NO: _____

1. Do not open this exam until you are told to begin.
2. This exam has 9 pages including this cover. There are 10 questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator. You are also allowed 2 sides of a 3 by 5 note card.
7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to make clear how you arrived at your solution.
8. Please turn **off** all cell phones.

PROBLEM	POINTS	SCORE
1	14	
2	8	
3	5	
4	6	
5	6	
6	12	
7	12	
8	15	
9	10	
10	12	
TOTAL	100	

1. (2 points each) Circle "True" or "False" for each of the following problems. Circle "True" only is the statement is *always* true. No explanation is necessary.

(a) $\log(\frac{1}{A}) = -\log(A)$.

True False

(b) If $f(x) = \pi^5$, then $f'(x) = 5\pi^4$.

True False

(c) The function $y = \frac{a}{b+ce^{-kt}}$ for $k > 0$ and a, b, c constants has a horizontal asymptote of $y = \frac{a}{c}$.

True False

(d) A degree 7 polynomial must have at least 1 real root but can not have more than 7 real roots.

True False

(e) $f'(a)$ is the tangent line of f at the point $(a, f(a))$.

True False

(f) If $f(x) = x^2$, then $f^{-1}(x) = \frac{1}{x^2}$.

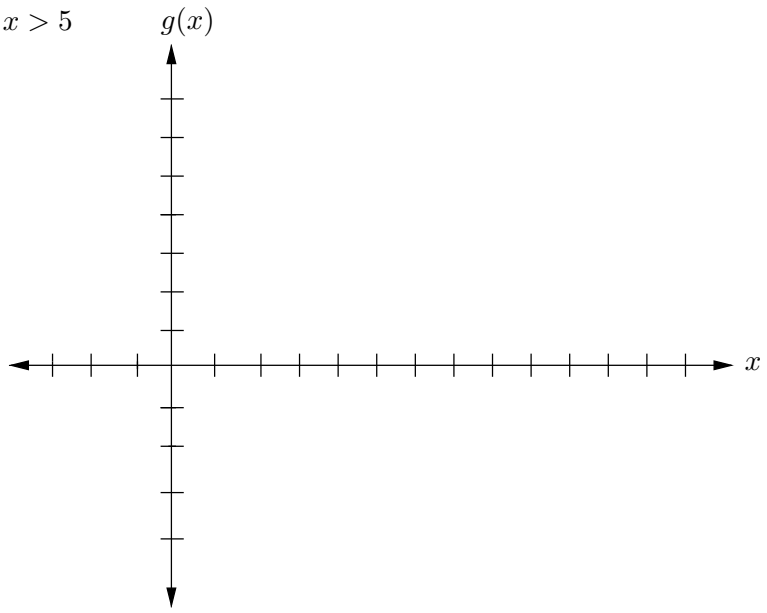
True False

(g) If $f''(a) = 0$, then the point $(a, f(a))$ is an inflection point of f.

True False

2.(8 points) On the axes below, sketch a graph of a single **function**, g, with **all** of the following properties.

- $g(0) = 2$
- $g'(x) > 0$ for $x < 5$
- $g''(x) > 0$ for $x < 0$
- $g''(x) < 0$ for $0 < x < 5$
- $\lim_{x \to 5^-} g(x) = 6$ and $\lim_{x \to 5^+} g(x) = 3$
- $g(5) = 4$
- $g'(x) = 0$ for $x > 5$



3. (1+1+3 points) Upon graduating from the university and landing your first big job, you decide to reward yourself for all the hard work and purchase a brand new sports car. The price of the sports car is $\$45,000$. The value of the car depreciates at the rate of 37% per year. Comprehensive insurance costs 10% of the car's value each year. For parts (a) and (b) circle the best choice.

(a) The value of the sports car is a Linear \ Exponential \ Both \ Neither function of time.

(b) The cost of the comprehensive insurance is a Linear \ Exponential \ Both \ Neither function of V, the value of the car.

(c) Write a function that gives the cost of the comprehensive insurance policy on the car after the t^{th} year.

4. (6 points) A circus is planning to visit your hometown. They claim to have a mummy that is 15,000 years old, but the citizens of your town are suspicious. The town council is given a sample of the mummy for a carbon-14 analysis. Your old high school science teacher is able to find that 33% of the mummy's carbon-14 remains. Using the fact that the half-life of carbon-14 is 5,730 years, determine the age of the mummy that the circus is bringing to town. [Show your work!]

5. (3 points each) The marketing department of Lay's Potato Chips decides to do a study on the number of chips a person craves as a function of the number of chips already eaten by that person. Their function turns out to be the rational function

$$C(x) = \frac{2x^2}{(x-1)^2},$$

where $C(x)$ is the number of additional chips a person who has eaten x chips is still craving to eat.

(a) How is Lay's famous slogan "You can't eat just one" summarized by this equation?

(b) What is the horizontal asymptote of $C(x)$? What does this say about a person with an unlimited supply of chips who can't control his/her cravings?

6. (12 points) For this problem f is differentiable everywhere.

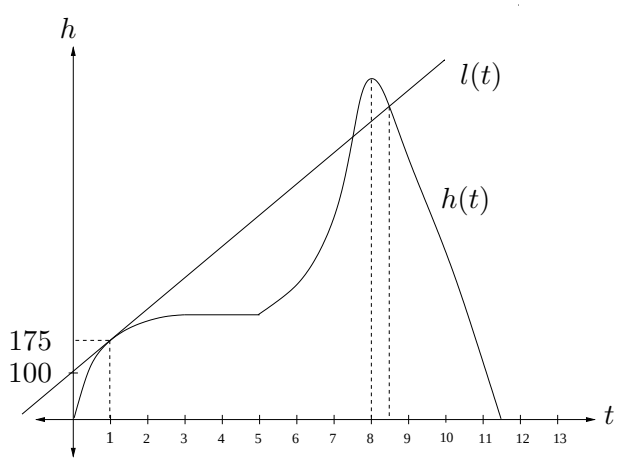
(a) Let $g(x) = f(x-2)$. Describe the graph of $g(x)$ in terms of the graph of $f(x)$.

(b) If $f'(1) = 6$, what is $g'(3)$? Don't do any calculations here, use the geometry of the situation from part (a) to arrive at your answer.

(c) State the limit definition of the derivative for the function f.

(d) Let $j(x) = f(x) + 10$. Use the limit definition of the derivative to calculate the derivative of j in terms of the derivative of f.

7. (12 points) The graph below gives a rock climber's height as a function of time as he climbs a small mountain. The height is measured in feet and the time is measured in hours. The line $l(t)$ gives the tangent line to $h(t)$ at time $t = 1$.



(a) For which time(s), if any, is the climber stopped?

(b) Does the climber speed up or slow down over the first three hours?

(c) What is the climber's rate of ascent 1 hour into the climb?

(d) What is the climber's height after 8.5 hours?

(e) If the maximum height the climber reaches is 800 feet, what is his average rate of ascent over the last 3.5 hours of his trip (i.e., for $8 < t < 11.5$)?

8. (3 points each) A study is published by a group of researchers at a prominent university that gives a person's expected annual salary after 10 years of work as a function, f, of the total amount of money that that person spent on college tuition. (The group counts loans, scholarships, family contributions, etc., as tuition that a person pays.) The tuition and salary are both measured in thousands of dollars.

(a) What does the statement $f(5) = 20$ mean in practical terms?

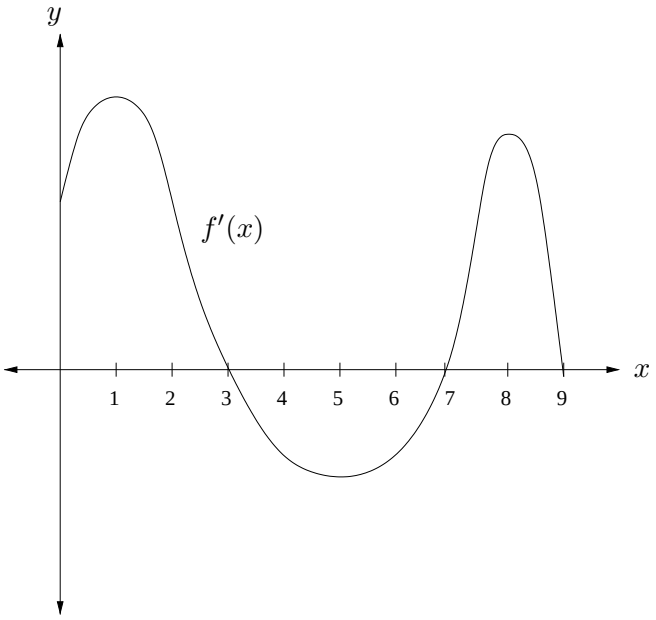
(b) What does $f^{-1}(50) = 20$ mean in practical terms?

(c) What do high-priced private schools hope is true about the sign of f'? Explain.

(d) What does the statement $f'(35) = 3$ mean in practical terms?

(e) Suppose you are trying to pick a college and your only concern is your expected salary after 10 years of work. If one of the schools you are considering will cost you $80,000$ in tuition and $f'(80) = -0.5$, should you choose a more expensive, less expensive, or that particular school? Justify your answer.

9. (10 points) The graph of $f'(x)$ (i.e., the *derivative* of f) is given below. Use the graph to answer the following questions:



(a) For which intervals is f increasing?

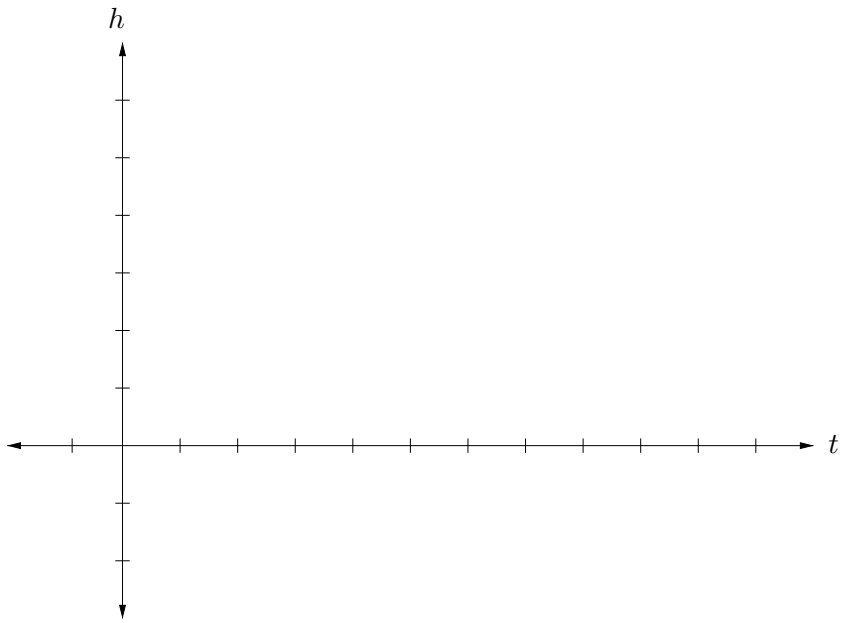
(b) For which intervals is f'' negative?

(c) For which value(s) of x (if any) does f have a local maximum?

(d) For which values of x (if any) does f switch from concave up to concave down?

10. (4 points each) At a nearby elementary school the seats of the swing set sit 2 feet off the ground when at rest. While observing a child swing, you note that the seat reaches a maximum height of 5 feet from the ground when the child swings without the aid of pushing from an adult. It takes the child 4 seconds to travel between successive maximum heights. (One is achieved while swinging forward, one while swinging backwards.)

(a) Sketch a graph of the seat's height above the ground (in feet) as a function of time (in seconds) on the axes provided below. Assume that at $t = 0$ the child is at her maximum height, and that she reaches the same maximum height each swing through. Be sure to label the axes carefully!



(b) Write a trigonometric equation describing the height of the seat as the child swings back and forth.

(c) At which time(s) during the first 4 seconds of motion is the height of the seat changing most rapidly?