MATH 115 — FIRST MIDTERM EXAM

February 8, 2005

NAME: ____

INSTRUCTOR: ______________________ SECTION NO: __________

1. Do not open this exam until you are told to begin.
2. This exam has 8 pages including this cover. There are 8 questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator. You are also allowed 2 sides of a 3 by 5 note card.
7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to make clear how you arrived at your solution.
8. Please turn **off** all cell phones and other sound devices, and remove all headphones.

PROBLEM	POINTS	SCORE
1	12	
2	12	
3	16	
4	15	
5	8	
6	5	
7	14	
8	18	
TOTAL	100	

1. (2 points each) Circle "True" or "False" for each of the following problems. Circle "True" only if the statement is *always* true. No explanation is necessary.

(a) Every continuous function is differentiable.

True False

(b) If $f'(x) > 0$ for all x in the interval (a, b), then f is increasing on the interval (a, b).

True False

(c) By definition, the instantaneous velocity is equal to a difference quotient.

True False

(d) Every rational function has a vertical asymptote.

True False

(e) If a function is not continuous at a point, then it is not defined at that point.

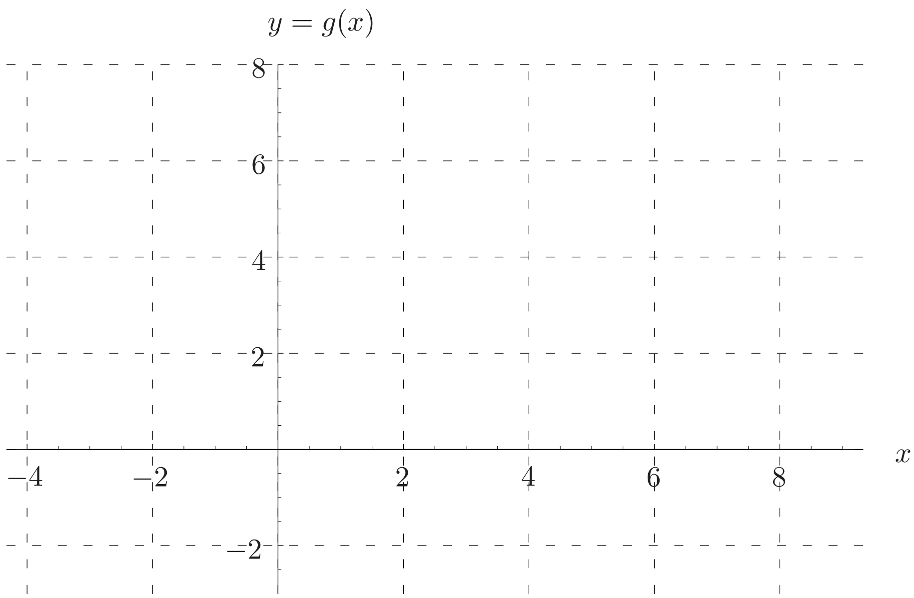
True False

(f) If a function f is decreasing on an interval, then f' is decreasing on that interval.

True False

2. (7+2+3 points) **(a)** On the axes below, sketch a graph of a single, continuous, differentiable **function**, g, with **all** of the following properties.

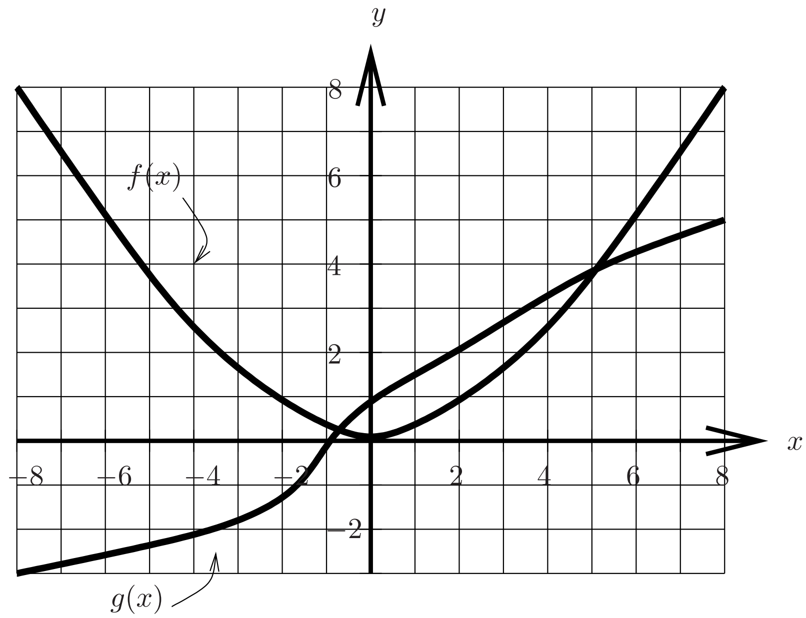
- $g(0) = 2$
- g' is negative for $x < 0$ and $x > 4$
- g is increasing for $0 < x < 4$
- g'' is positive for $x < 3$
- $g'(4) = 0$
- $g(x) \to 5$ as $x \to \infty$



(b) What is $\lim_{x \to -\infty} g(x)$?

(c) If $g'(1) = 1/2$, is it possible to have $g'(2) = 1/4$? Explain.

3. (16 points) The graphs of two functions, f and g are given in the figure below.



Use the graphs to estimate or answer each of the following.

(a) $f(g(2))$ _______________

(b) $3g(-4)$ _______________

(c) $g(f(2))$ _______________

(d) $g(2) \cdot f(6)$ _______________

(e) If $h(x) = f(x-1)$, how is the graph of h related to the graph of f?

(f) If $j(x) = g(-x)$, how is the graph of j related to the graph of g?

(g) The **sign** of $f'(-4)$ is _______________.

(h) The **sign** of $f''(-4)$ is _______________.

4. (6+2+3+4 points) For Valentine's Day last year, Hannah brought her officemates chocolate. As the day progressed, the amount of chocolate remaining in the office decreased. After two hours, there were only 5 pounds of chocolate remaining, and after seven hours, there was only 1 pound left.

(a) Assuming that the amount of chocolate in the office decreased linearly, write an equation for the amount of chocolate, c in pounds, left after t hours.

(b) How much chocolate did Hannah bring to the office?

(c) What is the practical interpretation of the slope of your linear function in the context of this problem.

(d) Now, assume instead that the amount of chocolate left at time t was represented by the exponential function $C(t) = ab^t$, find a and b, and express your answer as a function. [Do not assume that this function indicates the same beginning amount of chocolate as in part (a). Use the data given in the original statement of the problem to determine a and b.]

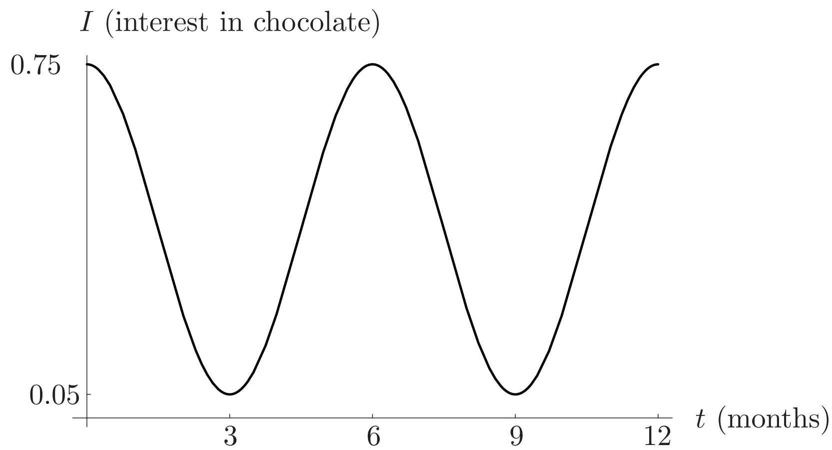
5. (8 points) After a dementor attack, Harry, the wizard, eats chocolate in order to feel better. When a wizard eats chocolate, the chocolate enters their bloodstream instantaneously and the body metabolizes and eliminates it from the bloodstream at the rate of 20% per hour.

(a) If Harry ate 1/2 pound of chocolate, write a formula for the amount of chocolate, Q (in pounds), remaining in his bloodstream t hours after he ate the chocolate.

(b) If Harry's chocolate level in his bloodstream becomes lower than 0.2 pounds, he will go into shock. What is the maximum amount of time, t, that he can wait before eating more chocolate? Show your work.

6. (5 points) Let $f(x) = \sin(3x^2)$. Use the **definition** of the derivative to express $f'(2)$ as a limit. You do **not** need to simplify your expression or try to approximate $f'(2)$.

7. (14 points) Kevin's interest in chocolate fluctuates during the year. His girlfriend works at a chocolate factory, and sometimes he gets a chocolate overload. Even at the best of times, he wouldn't gauge his level of "chocolate interest" as more than, say, a 75% interest. Assume that Kevin's interest in chocolate is given by the graph of $I(t)$ shown below, where t is in months and $t = 0$ is January 1st, 2004.



(a) Assuming I is a trigonometric function, find a formula for I in terms of t.

(b) List all months of 2004 in which Kevin's interest in chocolate was increasing.

(c) For what value(s) of t during 2004 was Kevin's interest in chocolate increasing the fastest?

8. (3 points each) Sarah decided to run a marathon. However, she started off way too fast and so her speed decreased throughout the race. Below is a table showing how many miles she had run at time t minutes since the beginning of the race.

time (min)	30	60	90	120	150	180	210	240
distance (miles)	5	9	12.5	15.5	18.5	21	23.25	25.2

Let s be the function such that $s(t)$ is Sarah's distance from the starting line t minutes after the race began.

(a) What is the practical interpretation of $s'(120)$ in the context of this problem?

(b) Estimate $s'(120)$.

(c) What is the practical interpretation of $s^{-1}(14)$ in the context of this problem?

(d) Estimate $s^{-1}(14)$.

(e) What does the derivative of $s^{-1}(P)$ at $P = 14$ represent in the context of this problem?

(f) Estimate the derivative of $s^{-1}(P)$ at $P = 14$.