MATH 115 — Second MIDTERM EXAM

March 29, 2005

NAME: ____

INSTRUCTOR: ______________________ SECTION NO: ________

1. **Do not open this exam until you are told to begin.**
2. This exam has 8 pages including this cover. There are 8 questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator. You are also allowed two sides of one 3 by 5 note card.
7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to make clear how you arrived at your solution.
8. Please turn **off** all cell phones and other sound devices, and remove all headphones.

PROBLEM	POINTS	SCORE
1	8	
2	10	
3	16	
4	12	
5	14	
6	10	
7	10	
8	20	
TOTAL	100	

1 . (8 points) The following table gives values of a continuous, differentiable function f' (i.e., the derivative of f). The statements below the table concern f. For each answer, give the smallest interval that is indicated by the table.

x	-4	-3	-2	-1	0	1	2	3	4
$f'(x)$	3	4	3	2	-1	-7	-2	4	6

(a) The function f has a local minimum between $x =$_____ and $x =$_____.

(b) The function f has a local maximum between $x =$_____ and $x =$_____.

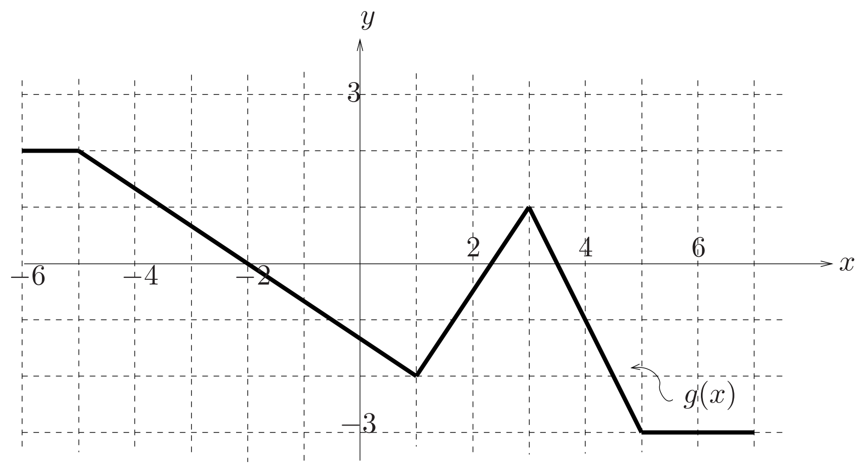
(c) The function f has an inflection point between $x =$_____ and $x =$_____. (There is more than one possible answer here.)

2. (10 points) Let g be a function such that $g(2) = 4$ and whose derivative is known to be $g'(x) = \sqrt{x^2 + 2}$.

(a) Use a linear approximation to estimate the value of $g(1.95)$. Show your work.

(b) Do you think your estimate in part (a) is an overestimate or an underestimate? Explain.

3. (16 points) Some values for a differentiable function f are given in the table below, and the graph of $y = g(x)$ on the interval [-6,7] is given in the figure below. Do not assume any information about f or g other than what is given.



x	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	0.5	4	7.5	10	9	5	0	3	9
$f'(x)$	3	4	3	2	-1	-7	-2	4	6

Use the table and the graph to find the following, if possible. If any information is missing, explain *clearly* what is missing. Show your work.

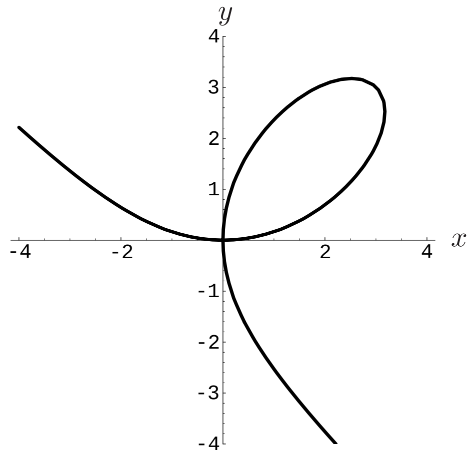
(a) Find $h'(4)$ if $h(x) = g(x)f(x)$.

(b) Find $h'(4)$ if $h(x) = g(f(x))$.

(c) Find $h'(-2)$ if $h(x) = 4\sin(g(x)) - \pi$.

(d) Find $h'(1)$ if $h(x) = (g(x))^2$.

4. (12 points) An example of Descartes' folium, shown in the picture below, is given by $x^3 + y^3 = 6xy$.



(a) Show that the point (3,3) is on the graph.

(b) Find the equation of the tangent to the graph at the point $(3, 3)$. Show your work.

(c) For what value(s) of x (if any) will the tangent to this curve be horizontal? [You do not need to solve for both x and y–just show x in terms of y.] Show your work.

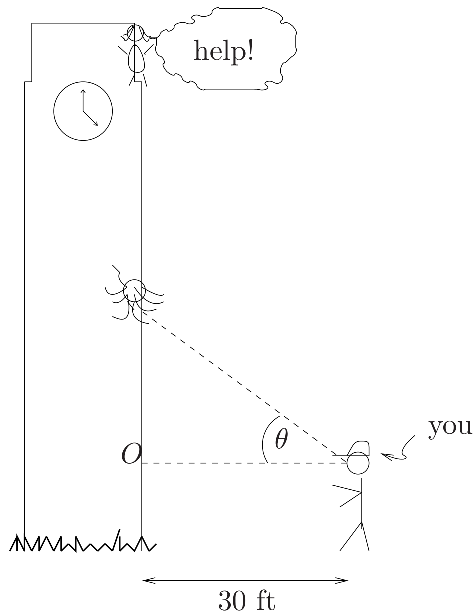
5. (14 points) A family of functions is given by $r(x) = \dfrac{a}{x}e^{bx}$ for a, b, and $x > 0$.

(a) For what values of a and b does the graph of r have a local minimum at the point $(4, 5)$? Show your work and **all supporting evidence** that your function satisfies the given properties.

(b) Write an explicit formula for $r(x)$. Circle your answer.

(c) Is the graph of r concave up or down for $x > 0$? Explain using arguments based on calculus–not only from a graph.

6. (10 points) Dr. Octopus is holding Mary Jane Parker hostage at the top of Burton Tower. Spiderman decides to climb the bell tower to try and rescue Mary Jane. Suppose you are standing 30 ft. away from the base of the tower watching Spiderman as he climbs. Let θ be the angle between the line of your horizon and your line of sight to Spiderman. The picture below may help you. [Picture not to scale.]



(a) Find a formula for the rate of change of Spiderman's distance from the point O with respect to θ.

(b) If the distance from point O to Mary Jane is 200 ft. and Spiderman is climbing at a constant 8 ft/sec, what is the rate of change of θ with respect to time when Spiderman reaches Mary Jane?

7. (10 points) The University has made an agreement with the Student Government Association to sell more student season football tickets. The tickets will cost \$150 each for the first 20,000 tickets. After 20,000 have sold, students will sign up for tickets. For each additional student (over 20,000) that signs up, the season price will be reduced by \$0.01 (yes, one cent) per student. A maximum of 35,000 total student tickets will be set aside. Students may sign up for tickets until August 20th. Effective on August 21st, the additional students may pick up their tickets at the reduced rate that has been determined by the number of students who had signed up by August 20th.

(a) What total number of student sales maximizes the university's revenue from student season football tickets? Show your work. [For full credit, you must show the function(s) you use for this problem. Just plugging numbers into a table will not suffice. In addition, show evidence of the use of calculus to find your answer–not merely a graph. State clearly what any variables in your function(s) represent.]

(b) What is the maximum revenue from student season ticket sales (based on this problem)?

(c) How many additional (i.e., over 20,000) season tickets would be optimal financially from the students' point of view? (Consider here only the students over the initial 20,000.) Explain.

8. (20 points) For each of the following, circle *all* correct answers. In each case, there may be more than one item which is correct.

(a) The function f' is continuous everywhere and changes from negative to positive at $x = a$. Which of the following *must* be true?

- a is a critical point of f.
- $f(a)$ is a local maximum of f.
- $f(a)$ is a local minimum of f.
- $f'(a)$ is a local maximum.
- $f'(a)$ is a local minimum.

(b) A function g is defined on all points of a closed interval. Which of the following *must* be true?

- g must have both a global maximum *and* a global minimum.
- g is differentiable on the interval.
- g has no critical points.
- g is continuous on the interval.
- None of the above statements *must* be true.

(c) For the graph of a cubic polynomial $ax^3 + bx^2 + cx + d$, $(a > 0)$, the signs of $f'(0), f''(0)$ and $f'''(0)$ (respectively) could be which of the following? (Circle all that are possible.)

- $-, 0, +$
- $-, 0, -$
- $+, +, +$
- $-, +, -$
- $+, -, +$

(d) The graph of $y = h(x)$ has a local max at $x = 3$ on the closed interval [0,5]. Which of the following *must* be true?

- $h'(3)$ is equal to zero or $h(3)$ is an end point.
- h has a critical point at $x = 3$.
- $h''(3)$ is positive.
- $h''(3)$ is negative.
- None of the statements *must* be true.

(e) Which of the following ***cannot*** be computed using L'Hopital's rule?

- $\lim_{x\to 0}(\sin x/x)$
- $\lim_{x\to 0}(\cos x/x)$
- $\lim_{x\to 0}(x/\sin x)$
- $\lim_{x\to \infty}(x/e^x)$
- $\lim_{x\to \infty}(\sin x/x)$