MATH 115 — FINAL EXAM

April 25, 2005

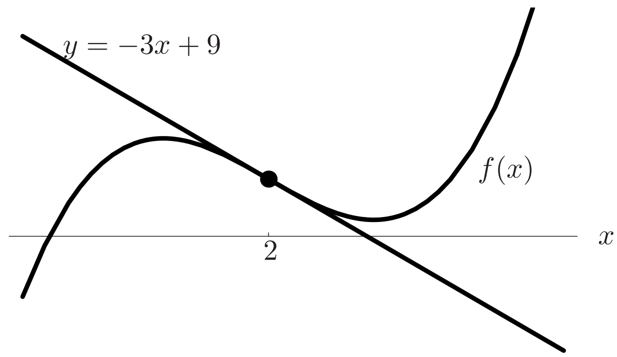
NAME: ____

INSTRUCTOR: ______________________ SECTION NO: __________

1. **Do not open this exam until you are told to begin.**
2. This exam has 9 pages including this cover. There are 9 questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator. You are also allowed two sides of one 3 by 5 note card.
7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to make clear how you arrived at your solution.
8. Please turn **off** all cell phones and other sound devices, and remove all headphones.

PROBLEM	POINTS	SCORE
1	12	
2	5	
3	5	
4	10	
5	12	
6	12	
7	17	
8	15	
9	12	
TOTAL	100	

1. (3+3+3+3 points) The figure below shows the tangent line approximation of $f(x)$ near $x = a$.



(a) What are a, $f(a)$, and $f'(a)$?

$a =$ ______ $f(a) =$ ______ $f'(a) =$ ______

(b) Estimate $f(2.1)$. Is this an overestimate or an underestimate? Why?

$f(2.1) \approx$ ______ is an ______________________ because

(c) Estimate $f(1.98)$. Is this an overestimate or an underestimate? Why?

$f(1.98) \approx$ ______ is an ______________________ because

(d) Would you expect your estimation for $f(2.1)$ or $f(1.98)$ to be more accurate? Why?

2. (5 points) Suppose $\int_4^9 (4f(x) + 7)dx = 315$. Find $\int_4^9 f(x)dx$.

3. (5 points) Use the Fundamental Theorem to determine the positive value of b if the area under the graph of $f(x) = 4x + 1$ between $x = 2$ and $x = b$ is equal to 11.

4. (2 points each–no partial credit) Suppose $\int_a^b f(x)dx = 2$ and $\int_a^b g(x)dx = 6$. Evaluate the following expressions, if possible. If the expression cannot be evaluated with what is given, simply indicate "Insufficient information." Assume that all functions are continuous on the interval $[a, b]$.

(a) $\int_a^b (g(x))^2 dx - (\int_a^b g(x)dx)^2$

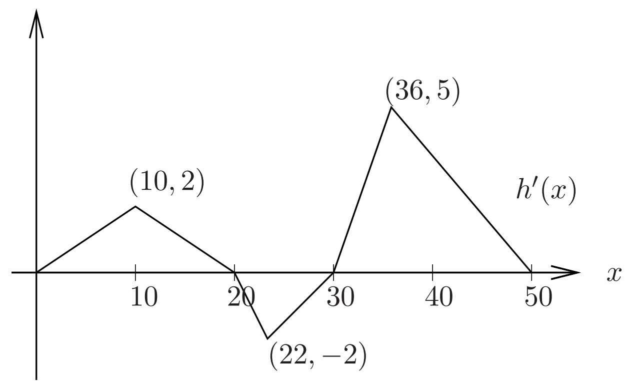
(b) $\int_a^a (h(x))dx$

(c) $\int_{a+2}^{b+2} f(x-2)dx$

(d) $\int_a^b (f(x)g(x))dx$

(e) $\int_b^a (g(x))dx$

5 . (12 points) Using the graph of h' in the figure below and the fact that $h(0) = -20$, sketch the graph of $h(x)$. Give the coordinates of all critical points, inflection points, and end points of h. Pay attention to the concavity of the graph.



6. (4+4+4 points) Harry Potter, Ron, and Hermione decide to attend the Wizard Fair. The newest ride at the fair, called **The Coil of Doom**^{TM}, is a spin-off on bungee jumping. Riders are attached to a special bungee cord which oscillates up and down. The riders' position above the ground, in feet, is given as a function of time, t, in seconds, by $y = y_0 cos(\omega t) + C$, with y_0, ω, and C constants.

(a) The riders board from a platform 15 feet above the ground, are pulled upward until, 6 seconds later, they reach a maximum height of 165 feet. In another 6 seconds, riders are back at the initial position. The cycle repeats for one minute, at which point the ride ends. Using this information, determine an explicit formula for y. [Show all constants in *exact* form.]

(b) Find formulas for the velocity and acceleration of the riders as a function of t.

(c) Show that the function y satisfies the equation $\frac{d^2y}{dt^2} + \omega^2 y = K$, where K is a constant. What is the value of K?

7. (17 points) At the Wizard Fair, there is a booth where wizards win Bertie Bott's Every Flavor Beans. To determine how many beans one gets, a contestant is given a string 50 inches long. From this string, contestants can cut lengths to form an *equilateral triangle* and a *rectangle whose length is twice its width.* The number of Bertie Bott's beans one wins depends on the combined areas of the triangle and rectangle. Harry, knowing calculus, goes immediately to work setting up a function, finding critical points, etc.

(a) Use your knowledge of calculus to determine the areas of the triangle and rectangle that will maximize the number of beans that Harry can win. Show your work.

(b) If the number of beans won is 9 times the combined area, what is the greatest number of beans a contestant can win?

8. (3 points each) Harry, Ron, and Hermione are all thrilled about their abundance of Bertie Bott's Every Flavor Beans; however, they prefer Chocolate Frogs to Bertie Bott's Beans. Luckily, at the wizard fair there is a booth where wizards are able to exchange Bertie Bott's Beans for Chocolate Frogs. The number of beans, N, needed to "purchase" F chocolate frogs is given by the function $N = C(F)$. Using *complete* sentences, give the practical interpretations of each of the following statements in the context of this problem.

(a) $C(3)$

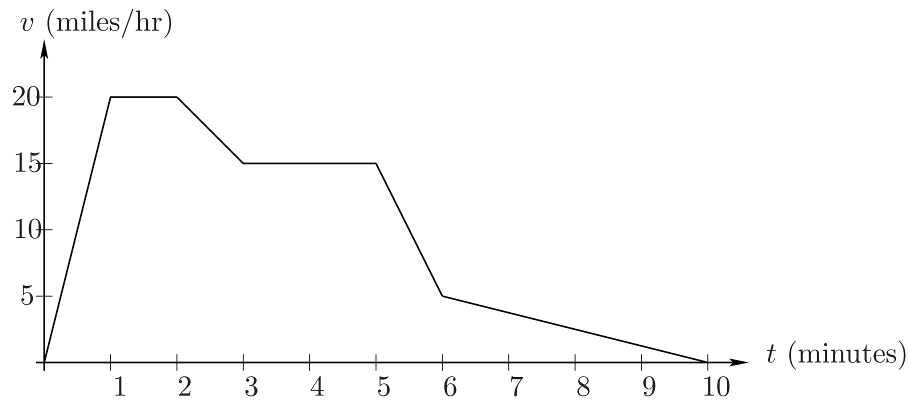
(b) $C'(3) = 18$

(c) $C^{-1}(91)$

(d) $(C^{-1})'(91) = 0.05$

(e) $\int_4^{10}(C'(F))dF$

9. (5+2+2+3 points) The three happy wizards leave the fair and go home to watch the Simpsons. In this episode, Homer needs to deliver Lisa's homework to her at school, and he must do so before Principal Skinner arrives. Suppose Homer starts from the Simpson home in his car and travels with velocity given by the figure below. Suppose that Principal Skinner passes the Simpson home on his bicycle 2 minutes after Homer has left, following him to the school. Principal Skinner is able to sail through all the traffic and travels with constant velocity 10 miles per hour.



(a) How far does Homer travel during the 10 minutes shown in the graph?.

(b) What is the average of Homer's velocity during the 10 minute drive?

(c) At what time, $t > 0$, is Homer the greatest distance ahead of Principal Skinner?

(d) Does Principal Skinner overtake Homer, and if so, when? Explain.