Math 115 – Second Midterm Exam

March 28, 2006

Name: ______________________________

Instructor: ______________________ Section Number: ______________

1. **Do not open this exam until you are told to begin.**
2. This exam has 9 pages including this cover. There are 8 questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator. You are also allowed two sides of a 3 by 5 notecard.
7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to show how you arrived at your solution.
8. Please turn off all cell phones and pagers and remove all headphones.

Problem	Points	Score
1	20	
2	10	
3	18	
4	6	
5	10	
6	12	
7	16	
8	8	
Total	100	

1. (20 points) A small company called Maple, Inc. is designing a fancy gift box with a square base. The box must have a volume of 3000 cm^3. The gift box has a lid which is to be made of a material that costs \$1 per square centimeter. The material for the sides of the box costs \$0.75 per cm^2, and the material for the bottom is \$0.80 per cm^2.

   (a) (10 pts.) What are the dimensions of the cheapest gift box the company can make?

   It turns out that Maple, Inc. also produces a cube-shaped wooden box to store jewelry. The cost of producing q of these boxes is given by

$$C(q) = 8600 + 0.0001(q - 80)^3(q + 90).$$

   (b) (3 pts.) What is the marginal cost when 80 boxes are made? Show your work.

(This problem continues on the next page.)

(This is a continuation of Problem 1).

(c) (3 pts.) The marginal cost of producing 95 of the cube-shaped jewelry boxes is about \$13 per box. Explain what this means in practical terms. (Your explanation should be understandable to someone who does not know calculus or economics language).

(d) (4 pts.) Let R and P denote, respectively, the revenue and the profit of Maple, Inc. from selling q of the cube-shaped jewelry boxes. Fill in the blank and circle the right choice in the paragraph below, as indicated.

If the profit P is maximized when 95 jewelry boxes are sold, then

$R'(95) =$ ____________ dollars per box *(fill in the blank)*, and $P''(95)$ must be

POSITIVE / NEGATIVE / ZERO *(circle the appropriate choice).*

2. (10 points) Suppose f has a continuous derivative whose values are given in the following table.

x	0	1	2	3	4	5	6	7	8	9	10
$f'(x)$	5	2	1	-2	-5	-3	-1	2	3	1	-1

(a) Using the data in the table, estimate x-coordinates of indicated critical points of f for $0 < x < 10$.

(b) For each critical point above, indicate if it is a local maximum of f, a local minimum, or neither.

(c) Approximate interval(s) between $x = 0$ and $x = 10$, if any, for which the data indicates that the graph of f is concave up?

(d) If $f(0) = 4$, approximate the value of $f(0.2)$.

3. (18 points)

(a) (2 pts) If $f(x) = ax^4 - x^3 + d$ $(a \neq 0)$ and f has a global maximum, what must be the sign of a? Explain.

(b) (4 pts) Determine all critical points of f.

(c) (4 pts) For what value of x does the maximum occur? Show your work.

(d) (4 pts) For what value(s) of x (if any) does f have inflection points?

(e) (4 pts) If $f(0) = 4$ and f has a critical point at $x = -\dfrac{1}{4}$, determine a formula for $f(x)$.

4. (6 points) Find the *exact* equation of the linear approximation to the curve $f(x) = 10e^{0.4x}$ having slope equal to 2.

5. (10 points) Find the *exact* coordinates of the point (x, y) where the tangent line to the graph of

$$y^3 - xy = -6$$

is vertical. You should start by differentiating the equation above implicitly with respect to x. Show step-by-step work.

6. (12 points) The functions $r = f(t)$ and $V = g(r)$ give the radius and the volume of a commercial hot air balloon that is being inflated for testing. The variables t and r are measured in minutes and feet respectively, while the volume V is measured in cubic feet. The inflation begins at $t = 0$.

Use the information on the tables below to answer questions (i)-(iii). (Question (iv) is independent of the tables.)

t	$f(t)$	$f'(t)$
0	c	d
30	b	x
60	a	z

r	$g(r)$	$g'(r)$
a	b	x
b	c	z
d	x	y

(i) (2 pts.) How fast is the radius of the balloon increasing initially?

____________________ ft/min.

(ii) (2 pts.) Assuming f is always increasing for $0 < t < 60$, how much time has ellapsed (since inflation began) when the radius is growing by z ft/min?

____________________ minutes.

(iii) (3 pts.) How fast is the volume of the balloon increasing a half hour after inflation began?

____________________ ft^3/min.

(iv) (5 pts.) *(This item is independent of the previous ones).* It turns out that the balloon's surface area increases with the radius by the formula

$$S = h(r) = 4\pi r^2$$

If the radius of the balloon increases linearly from 5 feet at a rate of 1.5 feet per minute, how fast is the balloon's surface area growing an hour after inflation began? Show your work.

7. (2 points each) Circle "TRUE" or "FALSE" for each of the following problems. Circle "TRUE" only if the statement is *always* true. No explanation is necessary.

(a) If $f(x)$ is increasing, then $f'(x)$ is increasing.

TRUE FALSE

(b) Suppose $f'(a) \geq f'(b)$ whenever $a \leq b$. Then f has no points of inflection.

TRUE FALSE

(c) If $f(x)$ is defined for all x, then $f'(x)$ is defined for all x.

TRUE FALSE

(d) If f and g are functions whose second derivatives are defined, then $(fg)'' = fg'' + f''g$.

TRUE FALSE

(e) If the radius of a circle is increasing at a constant rate, then so is the area.

TRUE FALSE

(f) If $f(x)$ has an inverse function, then the derivative of the inverse function is $1/f'(x)$.

TRUE FALSE

(g) If $f'(1) = -3.4$ and $g'(1) = 4.1$, then the function $h(x) = f(x) + g(x)$ is increasing at $x = 1$.

TRUE FALSE

(h) The graph of $y = xe^{-0.1x}$ has an inflection point at $x = 20$.

TRUE FALSE

8. (8 points) Below are the graphs of two functions f and g and their derivatives. Consider the function h defined by

$$h(x) = (f(x))^2 + (g(x))^2.$$

Find approximate values for $h(2)$ and $h'(2)$. [Show your intermediate calculations as well as your final answers below.]

- $h(2) \approx$ ______________________________

- $h'(2) \approx$ ______________________________