Math 115 – Final Exam

April 20, 2006

Name: ______________________________

Instructor: ______________________ Section Number: ______________

1. **Do not open this exam until you are told to begin.**
2. This exam has 9 pages including this cover. There are 8 questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator. You are also allowed two sides of a 3 by 5 notecard.
7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to show how you arrived at your solution.
8. Please turn off all cell phones and pagers and remove all headphones.

Problem	Points	Score
1	14	
2	12	
3	9	
4	12	
5	12	
6	12	
7	12	
8	17	
Total	100	

1. (14 points) Problems (a), (b) and (c) below are independent of each other.

(a) (5 pts.) Compute the linear approximation to $g(x) = 3\ln(x^2)$ near $x = 1$.

(b) (3 pts.) Write the limit definition of the derivative of the function $f(x) = e^x - e^{-x}$ at the point $x = a$. You do *not* need to simplify or attempt to compute the limit.

(c) (6 pts.) Assuming the following table accurately represents the behavior of the continuous function $s(x)$ over the interval $[0, 12]$, approximate the following:

[*NOTE*: the values in the table are for $s'(x)$, not $s(x)$].

x	0	3	6	10	12
$s'(x)$	-6	-3	0	1.2	17

(i) $s''(3)$

(ii) All intervals in $[0, 12]$ (if any) over which s is decreasing.

(iii) All intervals in $[0, 12]$ (if any) over which s is concave down.

2. (12 points) Problems (a) and (b) below are independent of each other.

(a) (7 pts.) In each case, calculate the value of the given integral expression. Where appropriate, you may assume that f is a differentiable function. *Your final answer should* **not** *contain any integral symbols and* **should be simplified** *as much as possible.* You may assume the symbols a, b and c represent constants. *Show your work!*

(i) $\displaystyle\int_a^b cf'(t)\,dt =$

(ii) $\displaystyle\frac{d}{dt}\left(\int_1^2 f(t)\,dt\right) =$

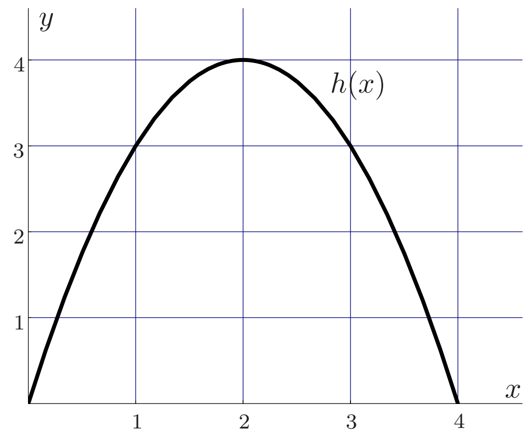
(iii) $\displaystyle\int_1^3 \left(c + \frac{t^3}{4}\right)\,dt =$

(b) (5 pts.) Assume now that f is a differentiable function of w, and that $w = w(x)$ is a differentiable function of x. Calculate the derivative indicated below. You may assume the symbol a stands for a constant. *Show your work.*

$\displaystyle\frac{d}{dx}\left(af(w) + xw^2\right) =$

3. (9 points) Problems (a) and (b) below are independent of each other.

(a) (6 pts.) The graph of a function $h(x)$ is given below.



- Numbers:

$A = h'(1), \quad B = h'(2), \quad C = h'(3), \quad D = h'(3.001), \quad E = \frac{h(3)}{3}, \quad F = \frac{h(3) - h(2)}{3 - 2}.$

- Write the numbers A–F from smallest to largest:

____

(smallest) (largest)

(b) (3 pts.) Consider the function $w(x)$ given by:

$$w(x) = \begin{cases} -x + 3, & 0 \le x < 1 \\ 2x, & 1 \le x \le 2 \end{cases}$$

Write the the numbers L, I, R (defined below) from smallest to largest.

- Numbers:

$L =$ Left-hand sum of w over [0,2] using 2 subdivisions

$I = \int_0^2 w(x)\ dx$

$R =$ Right-hand sum of w over [0,2] using 2 subdivisions.

- Ordered numbers:

____

(smallest) (largest)

4. (12 points) Problems (a) and (b) below are independent of each other.

   (a) (6 pts.) Suppose the function r gives the number of customers per day going to a new ice-cream store that just opened near campus. (Assume t is measured in days since the opening and that we are modeling the situation by a continuous function, r.) *IMPORTANT: The answers to (i) and (ii) should include clear units, and should be given using words understandable to someone who has never taken calculus.*

      (i) What does $\int_0^{20} r(t)\,dt$ represent?

      (ii) If each customer spends on average of \$3.50 in the store, what does the following expression represent?

$$\frac{3.5}{20}\int_0^{20} r(t)\,dt$$

   (b) (6 points) If the average value of the function $d(x) = 7/x^2$ on the interval $[1, c]$ is equal to 1, what is the value of c?

5. (2 points each) Circle "TRUE" or "FALSE" for each of the following problems. Circle "TRUE" only if the statement is *always* true. No explanation is necessary.

(a) $\displaystyle\int (1+y^2)\left(\frac{1}{y}\right)\, dy = (y+y^3)(\ln|y|) + C$

TRUE FALSE

(b) If f is any continuous function, then $\displaystyle\int_0^2 f(x)\, dx = \int_0^2 f(t)\, dt$.

TRUE FALSE

(c) If $\displaystyle\int_{-1}^2 g(x)\, dx + 6 = 10$ and g is an odd function, then $\displaystyle\int_1^2 g(x)\, dx = 4$.

TRUE FALSE

(d) If $\displaystyle\int_1^3 f(x)\, dx = 3$, then $\displaystyle\int_1^3 (3f(x)+2)\, dx = 11$.

TRUE FALSE

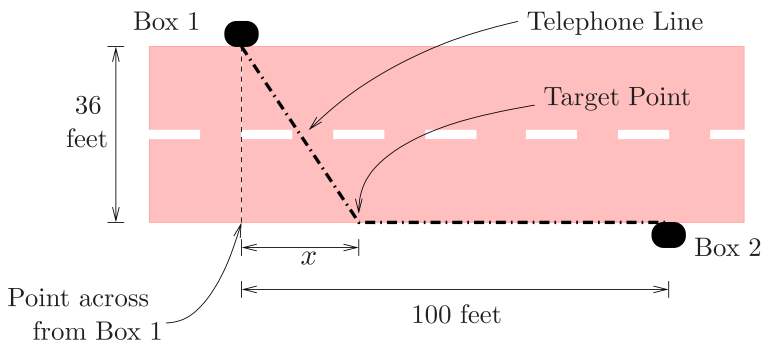
(e) If an object has constant nonzero acceleration, then the position of the object as a function of time is a quadratic polynomial.

TRUE FALSE

(f) If $f''(x)$ is continuous (over all the real numbers) and the graph of f has an inflection point at $x = p$, then $f''(p) = 0$.

TRUE FALSE

6. (12 points) A telephone installation crew must run a line underground between two junction boxes. Unfortunately, there is a 36 feet wide paved road between the two boxes, and one box is 100 feet down that lane from the other (see figure). It costs \$30 per foot to cut and repair the paved road, but only \$24 per foot to dig and refill along the side of the road. The crew will cut and repair the road to a point x feet from the point directly across from the first junction box, and then dig along the road the rest of the way. Determine the number of feet, x, from the point directly across from the first junction box which will minimize the cost of the installation.

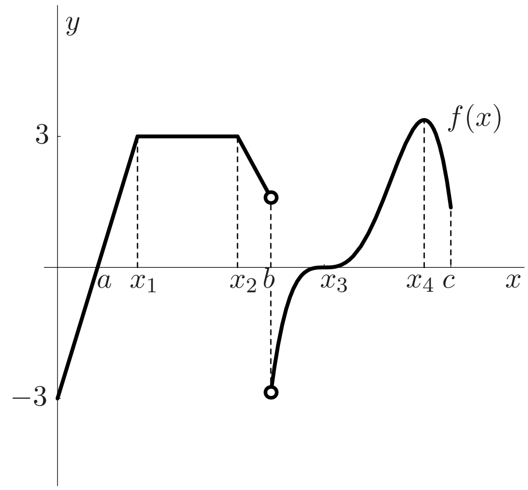


7. (12 points) You are heading due North across the Mackinac Bridge, and you sight your favorite fudge factory, located 1/4 mile due East of the end of the bridge. You might be fooling yourself, but the minute you sight the factory you are sure you can smell the chocolate fudge. You have to put the car on cruise control (at 55 mph) to resist the temptation to speed the rest of the way across the bridge. Show all work on parts (a) and (b) below.

   (a) If you are still 1.25 miles from the end of the bridge when you spot the factory, what is the distance (across the water) between your car and the factory?

   (b) If θ is the angle formed by a line between the factory and end of the bridge and the line from the factory to your car, how fast is θ changing at the time you spot the factory?

8. (17 points) Consider the graph of the function f given below. Your answers in parts (i) through (iv) may contain some of the constants a, x_1, x_2, b, x_3, x_4, or c.



(i) (2 pts.) Consider just the interval (b, c). Find all the x-values which are critical points of f on this interval (if any).

Critical points: ____.

(ii) (6 pts.) Determine the following and briefly justify your answers.

- The *value* of $\int_{x_2}^{x_1} f(x)\,dx$: ____________

JUSTIFICATION:

- The *sign* of $\int_{b}^{c} f(x)\,dx$: ____________

JUSTIFICATION:

(iii) (5 pts.) If: $F'(x) = f(x)$ and $F(0) = \pi$, estimate $F(x_2)$. Show step-by-step work.

(iv) (4 pts.) If F (from part (iii)) is a continuous function, determine the x-values of all the critical points of F on the interval $(0, c)$.

Critical points: ____.