

MATH 115 – FIRST MIDTERM EXAM

February 6, 2007

NAME: _____

INSTRUCTOR: _____ SECTION NUMBER: _____

1. **Do not open this exam until you are told to begin.**
2. This exam has 9 pages including this cover. There are 9 questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator. You are also allowed two sides of a 3 by 5 notecard.
7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to show how you arrived at your solution.
8. Please turn off all cell phones and pagers and remove all headphones.

PROBLEM	POINTS	SCORE
1	16	
2	9	
3	12	
4	13	
5	12	
6	16	
7	7	
8	7	
9	8	
TOTAL	100	

1. According to a survey by the U-M Transportation Research Institute, gasoline prices are projected to reach \$5.00 a gallon by the year 2020.
 - (a) (5 points) Assuming that the average gas price in 2007 is \$2.00 per gallon (yes, we know that is wishful thinking), find an exponential function, P , that models the average gas price t years after 2007. Show either an “exact” answer or at least 4 decimal places in your answer.
 - (b) (2 points) What is the *annual* percent change in the average gas price according to this model? (Show to at least one decimal place.)
 - (c) (2 points) What is the yearly *continuous* percent rate of change for this model? (Show to two decimal places.)
 - (d) (5 points) If, instead, gasoline prices grow linearly between 2007 and 2020, find a linear function, L , to model the price t years after 2007.
 - (e) (2 points) The survey indicates that prices may be \$4.00 per gallon eight years from now. Which of the two models best predicts this projection?

2. (9 points) Consider the following equations with a and b constants:

(i) $y^5 = ex$

(ii) $y - a^b = b(x - a)^{1/3}(x - a)^{2/3}$

(iii) $y - 2 = \sqrt{x^e}$

(iv) $\pi y = (9/13)^x$

Use the equations to answer the following. (One equation will not be used.)

(a) Which of the above can be written so that y is a linear function of x ?

Equation number _____

What is the slope of the function? _____

What is the y-intercept of the graph? _____

(b) Which of the above can be written so that y is an exponential function of x ?

Equation number _____

What is the initial value of the function? _____

What is the percent rate of growth/decay of the function? _____

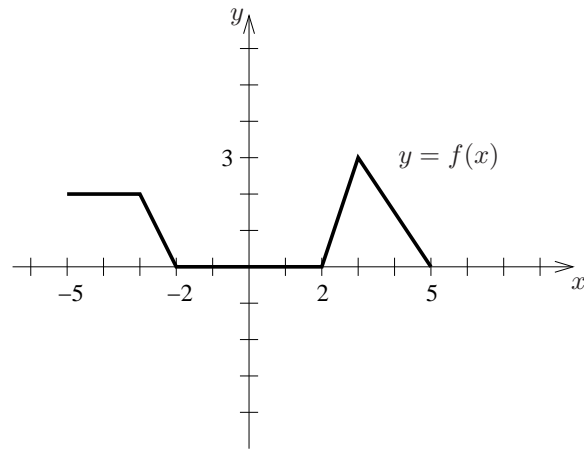
(c) Which of the above equations can be written as a power function of the form $y = kx^p$?

Equation number _____

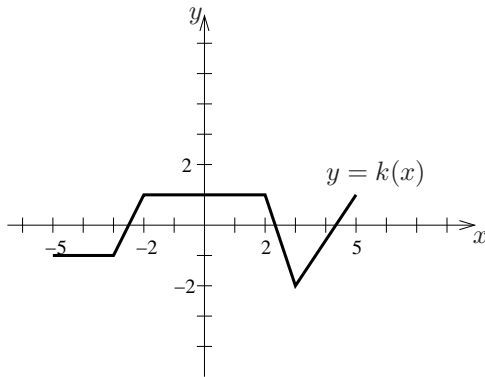
What is k ? _____

What is p ? _____

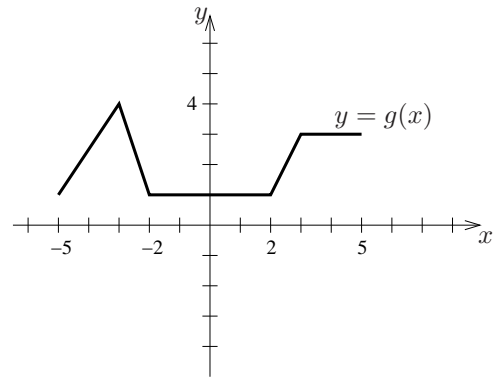
3. (12 points) The graph of $y = f(x)$ is given by the figure below.



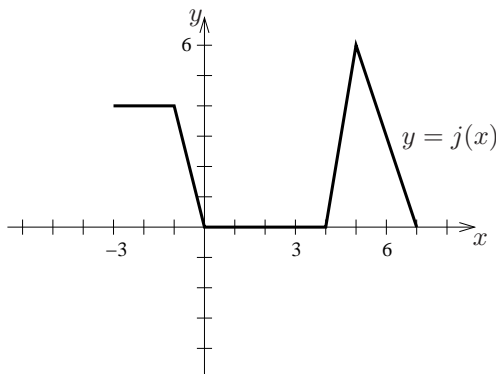
The graphs of the following functions are related to the graph of f . Determine a formula for each graph in terms of the function f .



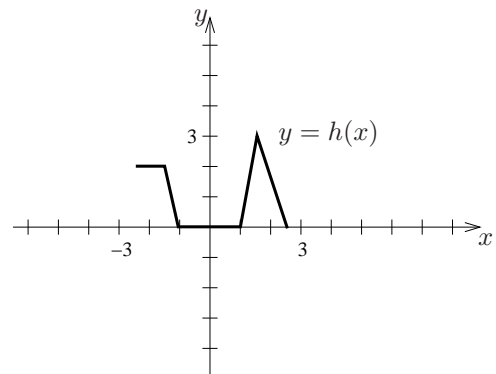
$$k(x) = \underline{\hspace{10em}}$$



$$g(x) = \underline{\hspace{10em}}$$



$$j(x) = \underline{\hspace{10em}}$$



$$h(x) = \underline{\hspace{10em}}$$

4. (13 points) Brian's favorite web site is woot.com. This site generally sells one item each day and records the number of sales during each hour of the daily special. On Thursday, Brian noted that the day's graph of sales looked sinusoidal. At 1:00 a.m., there were 70 items sold—and again at 3:00 p.m.. Between those hours, the sales went down (once) to a low of 20 items and then up (once) to a high of 120 items before the last 70 items were sold at 3:00 p.m..

(a) Determine a trigonometric function that would model sales, S , as a function of t in hours after 1:00 am, assuming that the graph Brian saw was sinusoidal.

(b) What is the period of your function?

The period is _____

(c) What is the amplitude of your function?

The amplitude is _____

(d) Approximately when were the sales increasing fastest?

5. (12 points) At woot.com the staff has become quite good at predicting the number of items that will be sold based on the brand name, reliability reports, the price, and the predicted popularity of the item. The maximum number of items, N , that they expect to sell during the entire sale period on a given day is a function of what they call the Max Sales Index, i , so $N = f(i)$, where the units of i are referred to as “points.”

(a) In the context of this problem, give a practical interpretation of $f(10)$.

(b) In the context of this problem, what is the practical interpretation of $f'(5) = 2500$?

(c) The number of Wooters (registered members of Woot.com) is currently over 500,000. Since there is not a mechanism for “un-registering,” and the membership has grown very quickly, assume that the number of Wooters, W in thousands, is an invertible function of time, t , in hours, $W = g(t)$. In this context, give a practical interpretation of $(g^{-1})'(200) = .05$

(d) Sometimes woot.com sells bags of junk, “like shopping blindfolded at the Dollar Store.” We can’t say the exact name here, so we’ll call them BoCs. Even these bags sell quickly on woot.com—typically in minutes. A recent BoC sale recorded the following data, where $s(t)$ gives the total number of BoC sales t minutes after the sale began. Use the data to estimate the $s'(10)$. Show your work.

time (minutes)	6	8	10	12	14	16	18
$s(t)$ (number of BoCs)	46	88	136	184	243	313	436

6. (16 points) State whether each of the following statements are TRUE or FALSE. For each statement, give an explanation. If the statement is false, give an example that shows a contradiction to the statement. If the statement is true, show why it is true. Examples may be formulas or graphs. Explain your reasoning.

(a) If $f'(x)$ is increasing, then $f(x)$ is also increasing.

(b) If $f(x) \neq g(x)$ for all x , then $f'(x) \neq g'(x)$.

(c) There is a function which is continuous on $[1,5]$ but not differentiable at $x = 3$.

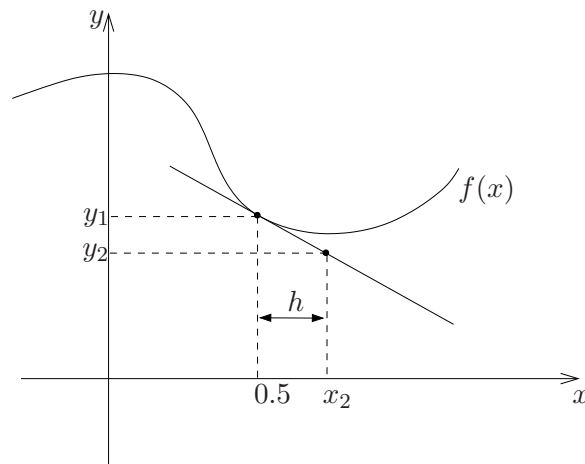
(d) If a function is increasing on an interval, then it is concave up on that interval.

7. (7 points) Use the function

$$g(x) = x^{\sin(x)}$$

to give the *limit definition* for $g'(2)$ [No need to simplify or approximate the limit.]

8. (7 points) The figure below shows $y = f(x)$ and a line tangent to f at $x = 0.5$. Given that $f(0.5) = 2$, $f'(0.5) = -3$, and $h = 0.1$, determine the values of y_1 , y_2 , and x_2 . [Note: x and y are different scales on the graph.]



$$y_1 = \underline{\hspace{2cm}}$$

$$y_2 = \underline{\hspace{2cm}}$$

$$x_2 = \underline{\hspace{2cm}}$$

9. (8 points) Cosmologists, through a technique best described as hocus pocus, measure a quantity $T(t)$, the temperature of the universe in degrees Kelvin (K), where t is in gigayears (Gyr) after the Big Bang. Suppose that, currently, $t = 13.6$, $T(13.6) = 2.4$, and $T'(13.6) = -12$.

[Note: A gigayear is 1 billion years, and the Kelvin temperature scale is an absolute temperature scale where the lowest possible temperature is defined as being zero Kelvin.]

- (a) For each of the following statements, state whether you agree or disagree with the conclusion and justify your reasoning.

(i) In the next billion years, the temperature of the universe will drop by approximately 12 degrees Kelvin.

(ii) In the next year, the temperature of the universe will drop by approximately $\frac{12}{1,000,000,000}$ degrees Kelvin.

- (b) Assume $T(t)$ is decreasing and does not change concavity on the domain $[13.6, \infty)$. Do you expect $T(t)$ to be concave up or concave down on the domain $[13.6, \infty)$? Justify your answer using physical reasoning.