

MATH 115 – FINAL EXAM

April 20, 2007

NAME: _____

INSTRUCTOR: _____ SECTION NUMBER: _____

1. **Do not open this exam until you are told to begin.**
2. This exam has 10 pages including this cover. There are 9 questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator. You are also allowed two sides of a 3 by 5 notecard.
7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to show how you arrived at your solution.
8. Please turn off all cell phones and pagers and remove all headphones.

PROBLEM	POINTS	SCORE
1	10	
2	12	
3	12	
4	12	
5	16	
6	12	
7	12	
8	8	
9	6	
TOTAL	100	

1. (2 points each, no partial credit) For the following statements circle True or False. Circle True only if the statement is *always* true.

(a) If y is differentiable for all x , then the value of $y'(x)$ is a unique number for each x .

True

False

(b) The only antiderivative of $\cos(x)$ is $\sin(x)$.

True

False

(c) For a continuous function f on the interval $a \leq x \leq b$, if the left-hand sum and the right-hand sum are equal for a given number of subdivisions, then they are equal to $\int_a^b f(x)dx$.

True

False

(d) For the continuous function f , if the units of t are seconds and the units of $f(t)$ are meters, then the units of $\int_0^1 f(t)dt$ are meter seconds.

True

False

(e) For any function f , if $\lim_{x \rightarrow 3^-} f(x) = a$ and $\lim_{x \rightarrow 3^+} f(x) = a$, then $f(3) = a$.

True

False

2. (12 points) Suppose that f and g are continuous functions and $\int_0^2 f(x)dx = 5$ and $\int_0^2 g(x)dx = 13$. Compute the following. If the computation cannot be made because something is missing, explain clearly what is missing.

(a) $\int_4^6 f(x-4)dx$

(b) $\int_{-2}^0 2g(-t)dt$

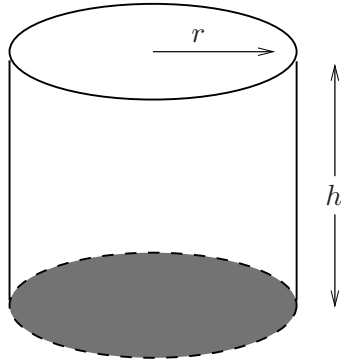
(c) $\int_2^0 (f(y) + 2)dy$

(d) $\int_2^2 g(x)dx$

- (e) Suppose that f is an even function. Compute the average value of f from -2 to 2 .

3. (12 points) Octavius, the giant octopus, escaped after all. He was being kept in a temporary tank near the harbor—apparently even less secure than his tank at the zoo. He managed to get into the bay, but the coast guard could keep track of him with a homing device that had been attached to Octavius at the zoo.
- (a) The coast guard station is 2 kilometers (2000 meters) down the beach from where Octavius entered the bay. If the octopus was moving directly away from the shore at a constant rate of 25 meters per minute, how fast was the distance between the coast guard and Octavius changing when the octopus was 200 meters from the shore?
- (b) At what rate is the angle formed by the beach and the line that gives distance from the coast guard station to Octavius changing when Octavius is 200 meters from the shore?

4. (12 points) The zoo has decided to make the new octopus tank spectacular. It will be cylindrical with a round base and top. The sides will be made of Plexiglas which costs \$65.00 per square meter, and the materials for the top and bottom of the tank cost \$50.00 per square meter. If the tank must hold 45 cubic meters of water, what dimensions will minimize the cost, and what is the minimum cost?



radius _____

height _____

cost _____

5. (16 points) Use the information given in the table below to calculate the indicated values. If a value cannot be determined, state explicitly what is missing. Assume that f and f' are continuous, and that the table is reflective of the behavior of f .

x	0	3	6	9	12
$f(x)$	30	20	13	8	5
$f'(x)$	-4	-3	-2	-1.5	-0.5

Determine the following and show your work (3 points each):

- (a) an approximate value for $f(3.1)$ using a local linearization

- (b) a left-hand sum with 4 subdivisions to approximate $\int_0^{12} f(x)dx$

- (c) the least number of subdivisions necessary to assure that the left- and right-hand approximations of $\int_0^{12} f(x)dx$ agree to within 1 unit

- (d) $\int_3^{12} f'(x)dx$

Explain your answers to the following (2 points each):

- (e) Do you expect your approximation for $f(3.1)$ from part (a) to be an overestimate or an underestimate?

- (f) Do you expect your left-hand approximation from part (b) to be an overestimate or an underestimate?

6. (12 points) Suppose f and g are differentiable functions with values given by the table below. **To receive full credit, for each part below first show the formula for the derivative in terms of x , and then find the requested value.**

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	6	9	14	-7
2	4	13	1	-11

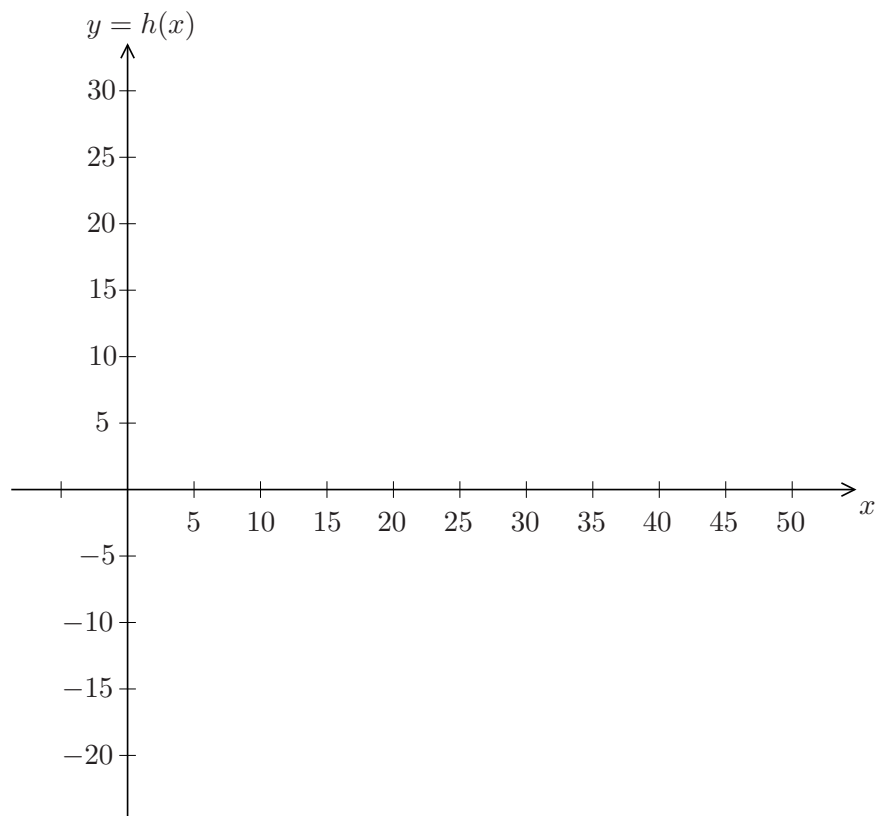
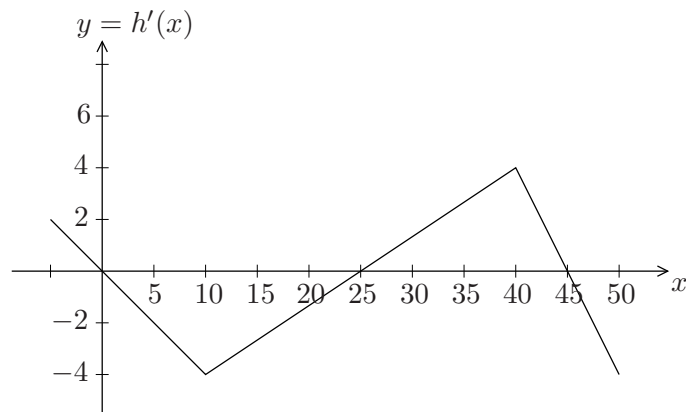
(a) If $h(x) = f(x)g(x)$, find $h'(1)$.

(b) If $j(x) = \frac{\ln(x)}{f(x)}$, find $j'(1)$

(c) If $d(x) = \sin(\cos(f(x)))$, find $d'(1)$.

(d) If $t(x) = g(x)g(2x)$, find $t'(1)$.

7. (12 points) The graph of the derivative, $h'(x)$, of a function h is given in the figure below. Given $h(10) = 10$, sketch the graph of $h(x)$ on the axes below the figure. Indicate the critical points and inflection points of h and give the coordinates of each of those points.



8. (8 points) Consider the family of functions of the form

$$f(x) = x + ax \ln(bx).$$

- (a) Under what conditions on a and b is the function f defined and concave down for positive values of x ? Explain. [Note: to receive credit, your work must be shown analytically, not just in the form of a graph.]

- (b) For what value(s) of x (if any), will f have a critical point?

9. (6 points)

(a) Write the *limit definition* of $g'(2)$ for an arbitrary differentiable function g .

(b) Use limit and summation notation to define $\int_a^b h(x)dx$ for an arbitrary continuous function h .