MATH 115 – SECOND MIDTERM

March 25, 2008

NAME: ______________________________

INSTRUCTOR: ____________________ SECTION NUMBER: ______________

1. **Do not open this exam until you are told to begin.**
2. This exam has **??** pages including this cover. There are **??** questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator. You are also allowed two sides of a 3 by 5 notecard.
7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to show how you arrived at your solution.
8. Please turn off all cell phones and pagers and remove all headphones.

PROBLEM	POINTS	SCORE
1	10	
2	11	
3	10	
4	15	
5	16	
6	12	
7	16	
8	10	
TOTAL	100	

1. (2 points each) For each of the following, circle all the statements which are **always** true. For the cases below, one statement may be true, **or** both **or** neither of the statements may be true.

   (a) Let $x = c$ be an inflection point of f. Assume f' is defined at c.

   - If L is the linear approximation to f near c, then $L(x) > f(x)$ for $x > c$.
   - The tangent line to the graph of f at $x = c$ is above the graph on one side of c and below the graph on the other side.

   (b) The differentiable function g has a critical point at $x = a$.

   - If $g''(a) > 0$, then a is a local minimum.
   - If a is a local maximum, then $g''(a) < 0$.

   (c) The derivative of $g(x) = (e^x + \cos x)^2$ is

   - $g'(x) = 2\,(e^x - \sin x)\,(e^x + \cos x)$.
   - $g'(x) = 2e^{2x} + 2\,(e^x \cos x - e^x \sin x)$.

   (d) A continuous function f is defined on the closed interval $[a, b]$.

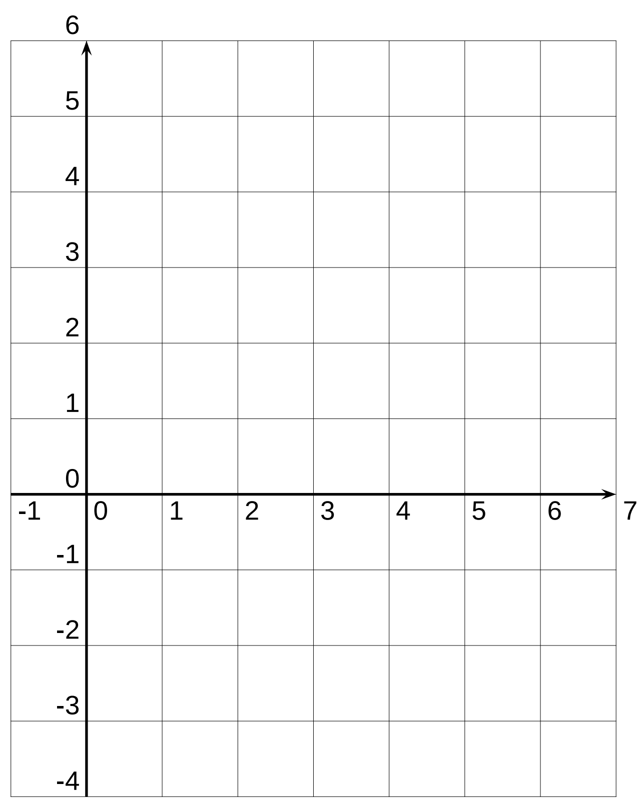
   - f has a global maximum on $[a, b]$.
   - f has a global minimum on $[a, b]$.

   (e) Consider the family of functions $e^{-(x-a)^2}$.

   - Every function in this family has a critical point at $x = 0$.
   - Some function in this family has a local maximum at $x = 2$.

2. (11points) Let $f(x)$ be a **continuous** function defined for all real numbers x. Sketch a possible graph of f, given that

- $f(4) = 2$;
- $f'(x) > 0$ **and** $f''(x) < 0$ for $x < 2$;
- $f'(2) = 0$ and $f''(2) = 0$;
- $f''(x) > 0$ for $2 < x < 4$;
- $f''(4) = 0$;
- $f''(x) < 0$ for $x > 4$;
- $f'(x) > 0$ for $2 < x < 5$;
- $f'(5) = 0$;
- $f'(x) < 0$ for $x > 5$.



3. (10 points) Suppose $f'(x)$ is a differentiable increasing function for all x. In each of the following pairs, circle the larger value. In each case, give a **brief** reason for your choice. (Assume that none of the values below are equal for this function and $\Delta x \neq 0$).

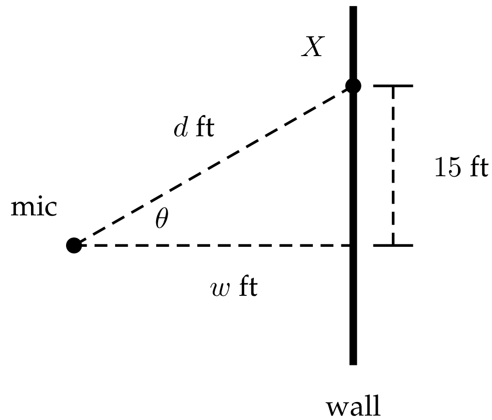
   (a) $f'(5)$ and $f'(6)$

   (b) $f''(5)$ and 0

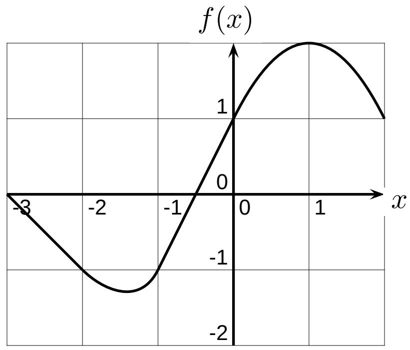
   (c) $f(5+\Delta x)$ and $f(5)+f'(5)\Delta x$

4. (15 points) Using calculus, find constants a and b in the function $f(x) = axe^{bx}$ such that $f(\frac{1}{3}) = 1$ and the function has a local maximum at $x = \frac{1}{3}$. Once you have found a and b, verify that your answer satisfies the given conditions. Show all work.

5. (16 points) A directional microphone is mounted on a stand facing a wall. The sensitivity S of the microphone to sounds at point X on the wall is inversely proportional to the square of the distance d from the point X to the mic, and directly proportional to the cosine of the angle θ. That is, $S = K\frac{\cos\theta}{d^2}$ for some constant K. (See the diagram below.) How far from the wall should the mic be placed to maximize sensitivity to sounds at X?



6. (12 points) The graph of a function f is shown below, together with a table of values for its derivative f'. Let $g(x) = f(f(x))$.



x	$f'(x)$
-3	-1
-2	-1
-1	2
0	2
1	0
2	-2

(a) (2 points) Find $g(-2)$

(b) (3 points) Find $g'(-2)$

(c) (3 points) Write an expression for $g''(x)$ in terms of f and its derivatives.

(d) (4 points) Suppose $f''(-1) = 2$. What is $g''(-1)$?

7. (a) (4 points) Show that the point $(x, y) = (3, -6)$ lies on the curve defined by $y^2 - x^3 - x^2 = 0$.

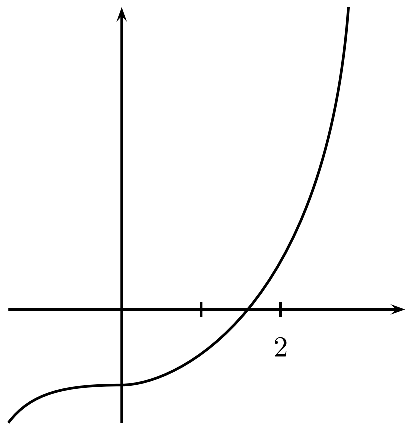
(b) (4 points) What is the equation of the tangent line to the curve at the point $(3, -6)$?

(c) (2 points) Consider the function $f(x) = x\sqrt{x+1}$. What is the domain of f?

(d) (6 points) Find all critical points, local maxima, and local minima of f. Which of the local maxima and minima are global maxima / minima? Show all work.

8. Let $f(x) = x^3 - a$, for $a > 1$ a constant. The graph of f is shown below.

   (a) (2 points) Label the numbers $-a$ and $\sqrt[3]{a}$ on the axes below.



   (b) (4 points) Find the equation for $L(x)$, the linear approximation to f near $x = 2$. Your equation will contain the constant a. Sketch the graph of $L(x)$ on the axes above.

   (c) (4 points) Use the function L to approximate 2.01^3.