MATH 115 – FINAL EXAM

April 22, 2008

NAME: ______________________________

INSTRUCTOR: ______________________ SECTION NUMBER: ______________

1. **Do not open this exam until you are told to begin.**
2. This exam has 10 pages including this cover. There are 10 questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator. You are also allowed two sides of a 3 by 5 notecard.
7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to show how you arrived at your solution.
8. Please turn off all cell phones and pagers and remove all headphones.

PROBLEM	POINTS	SCORE
1	8	
2	10	
3	8	
4	10	
5	12	
6	10	
7	12	
8	9	
9	6	
10	15	
TOTAL	100	

1. (2 points each) For each of the following, circle all the statements which are **always** true. For the cases below, one statement may be true, **or** both **or** neither of the statements may be true.

(a) Let $f(t) = t^2 + 2t$.

- $\frac{d}{dt} \int_0^1 f(t)\, dt = t^2 + 2t$.

- $\frac{d}{dt} \int_0^1 f(t)\, dt = 0$.

(b) Let $g(x)$ be continuous on the interval $[0, 1]$.

- The limit $\lim_{n \to \infty} \left(\sum_{k=1}^{n} g(x_k) \cdot \frac{1}{n} \right)$ exists.

- The limit $\lim_{h \to 0} \frac{g(0.5 + h) - g(0.5)}{h}$ exists.

(c) Suppose $\int_{-2}^{2} F(x)\, dx = 5$.

- F is not an odd function.

- For some c in the interval $[-2, 2]$, $F(c) > 1$.

(d) Suppose h is a differentiable function defined on $[a, b]$, with antiderivative H. Assume $h(t) > 0$ for all t in $[a, b]$

- h has either a local maximum or minimum (or both) on (a, b).

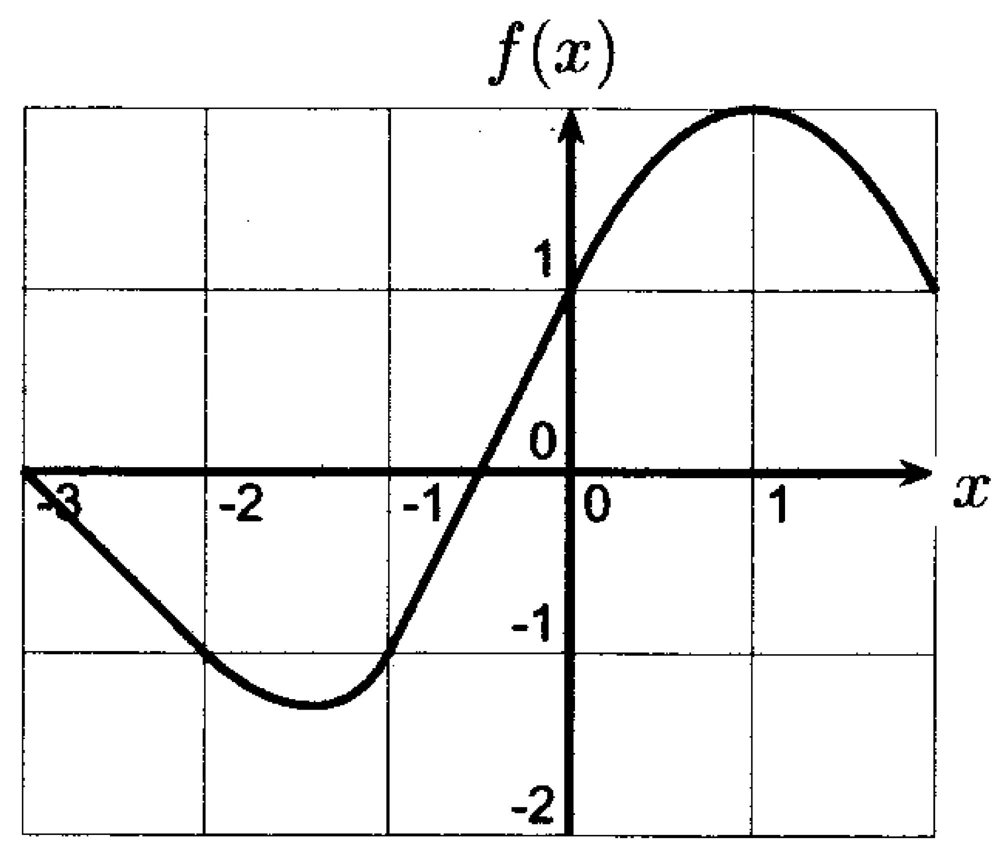
- H has a local maximum at b.

2. (10 points) A smokestack deposits soot on the ground with a concentration inversely proportional to the square of the distance from the stack. With two smokestacks 20 miles apart, the concentration of the combined deposits on the line joining them, at a distance x from one stack, is given by

$$S = \frac{k_1}{x^2} + \frac{k_2}{(20-x)^2}$$

where k_1 and k_2 are positive constants which depend on the quantity of smoke each stack is emitting. If $k_1 = 7k_2$, find the point on the line joining the stacks where the concentration of the deposit is a minimum.

3. (8 points) The graph of a function f is shown below.



Using the information given, write the following numbers in order from least to greatest:

$$\begin{aligned} A &= f'(1), \\ B &= \int_{-3}^{0} f(t)\,dt, \\ C &= \int_{-2}^{1} f(t+1)\,dt, \\ D &= \int_{-1}^{2} (f(t)+1)\,dt. \end{aligned}$$

_________ < _________ < _________ < _________

4. After seeing a Tigers game in Detroit, you get onto I-94 toward Ann Arbor at 10:00 pm. On I-94, you run into a few pockets of traffic. Glancing at the speedometer occasionally, you note your speed fluctuates as in the table below.

minutes after 10:00	0	5	10	15	20	25	30
speed (mph)	50	71	65	42	35	70	74

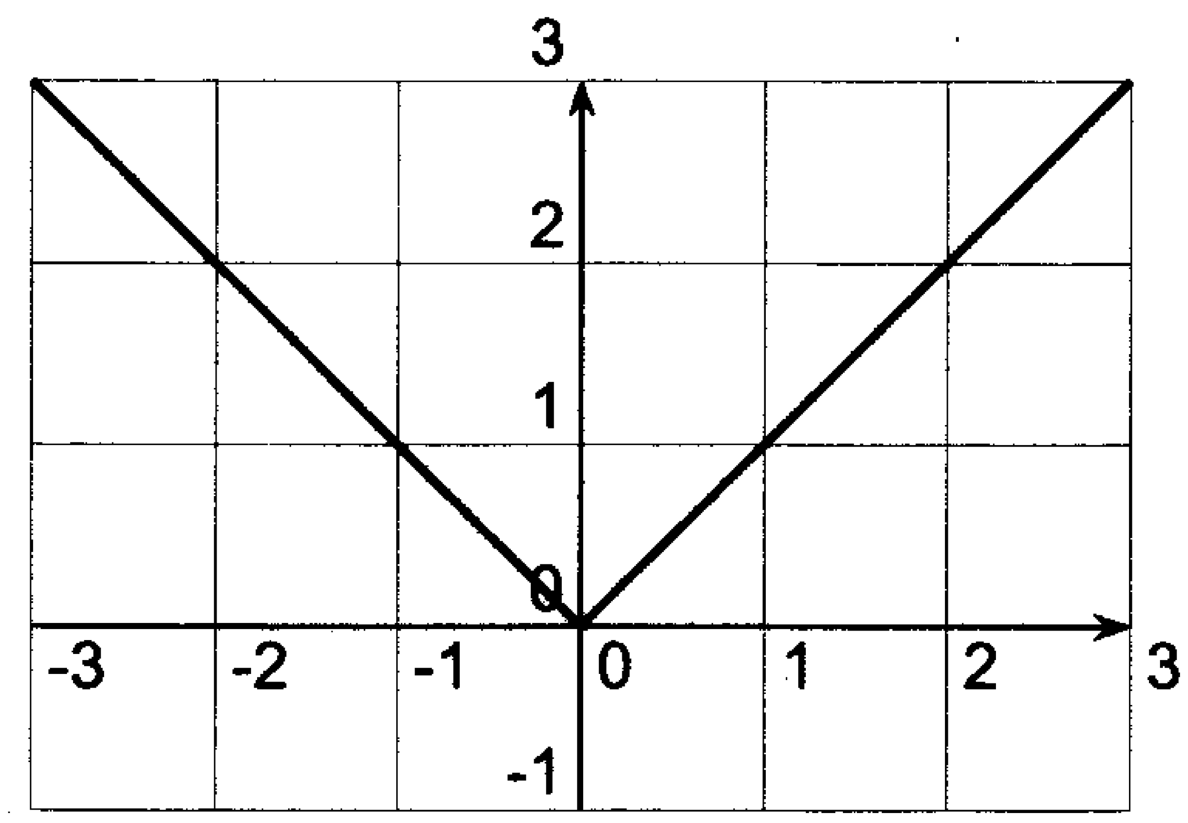
(a) (4 points) Using the information given, estimate your acceleration at 10:10. Be sure to include units.

(b) (6 points) Approximately how far from your entrance onto I-94 are you at 10:30? Be certain to show how you arrived at your approximation, and again, give correct units.

W08. INT

5. Let $f(x) = |x|$.

(a) (4 points) Find $\int_{-2}^{1} f(x)\,dx$ using geometry (i.e., areas). Show your work on the graph below and circle your numerical answer.



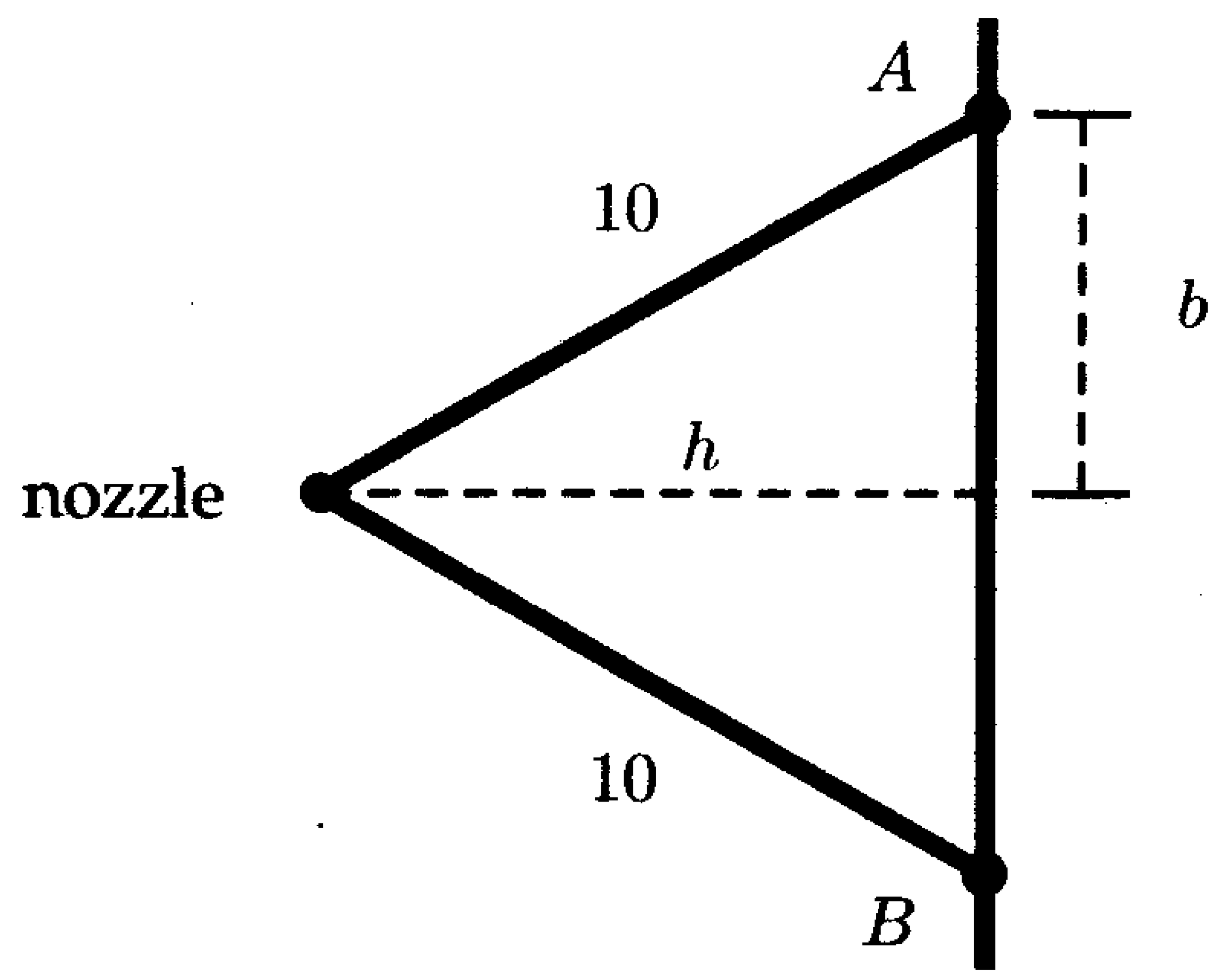
(b) (4 points) Find a formula for an antiderivative of $f(x)$, given the piecewise formula

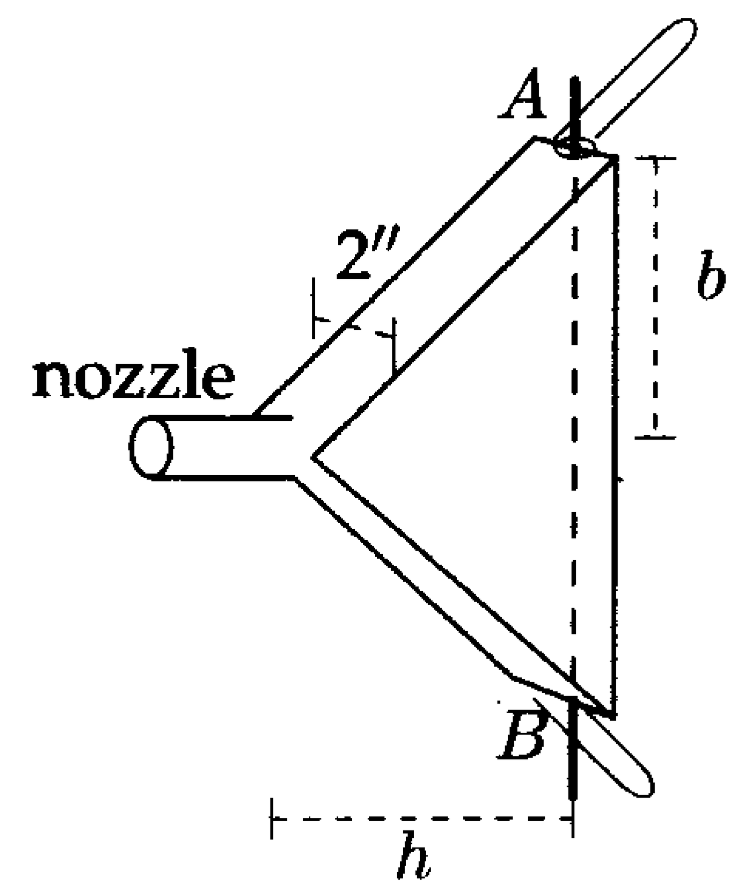
$$f(x) = |x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0. \end{cases}$$

(c) (4 points) Using the Fundamental Theorem and your answer to (5b), compute $\int_{-2}^{1} f(x)\,dx$.

W08 RR

6. (10 points) A bellows[1] has a triangular frame made of three rigid pieces. Two pieces, each 10 inches long, are hinged at the nozzle. They are attached to the third piece at points A and B which can slide, as shown in the diagrams below (the figure to the right shows a 3D sketch of the bellows; the figure to the left, a 2D sketch that may be specifically useful for solving the problem).



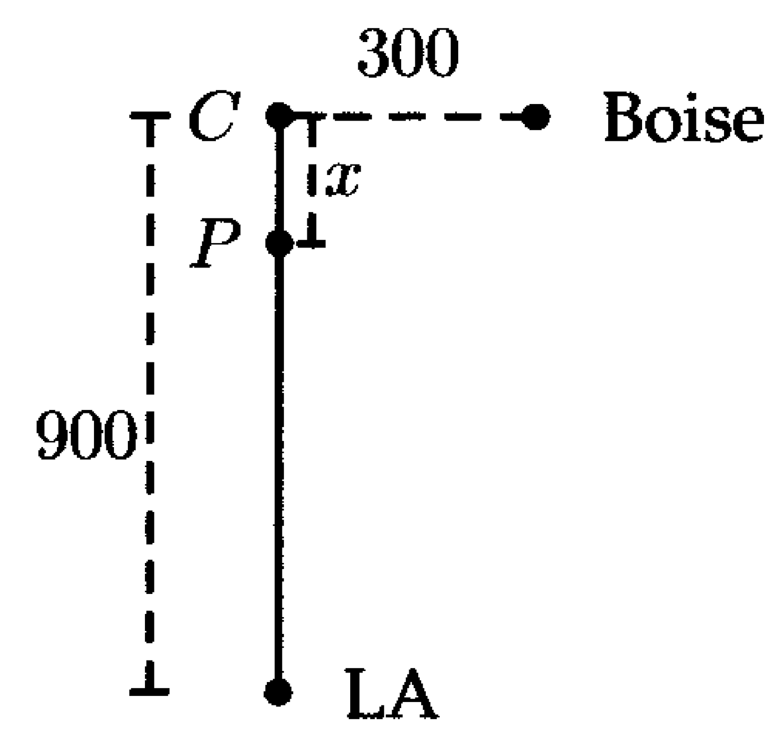


Each piece of the frame is 2 inches wide, so the volume (in cubic inches) of air inside the bellows is equal to the area (in square inches) of the triangular cross-section shown above times the height of 2. Suppose you pump the bellows by moving A downward towards the center at a constant speed of 3 in/s. (So B also moves upwards at the same speed.) What is the rate at which air is being pumped out when A and B are 12 inches apart? (So A is 6 inches from the center of the vertical piece of the frame.)

[1]A bellows is a device with a nozzle attached to a chamber; it is used to blow air out through the nozzle by reducing the volume of the chamber. In the bellows described here this is accomplished by moving the points A and B as indicated.

W08 OPT

7. (12 points) You decide to take your two-week vacation from your job at the Idaho Potato Company in Boise to drive on California's coastal highway and visit friends in Los Angeles.. Your plan is to fly a small plane to the coast, land at point P, and drive a rented car from there to LA. Assume the coast highway is straight, that C is the point on the coast directly west of Boise, and the distances (in miles) are as in the diagram below. (That is, ignore whatever actual experience you may have of that road!)



You want to minimize gas costs: it costs 30 cents per mile to fly, and 10 cents per mile to drive. Set up the function (of x) which describes the cost of gas for the trip, and find the position P where you should land to minimize the cost.

W08 INT

8. (3 points each) Let

- $\int_a^b f(x)dx = 8$, and $\int_a^b (f(x))^2 dx = 12$,
- $\int_a^b g(t)dt = 2$, and $\int_a^b (g(t))^2 dt = 3$.

Evaluate the following integrals, if the value can be determined. If there is information missing, clearly state what is missing.

(a) $\int_a^b (f(x) + g(x))dx$

(b) $\int_a^b cf(z)dz$, for c a constant

(c) $\int_a^b ((f(x))^2 - g(x^2))dx$

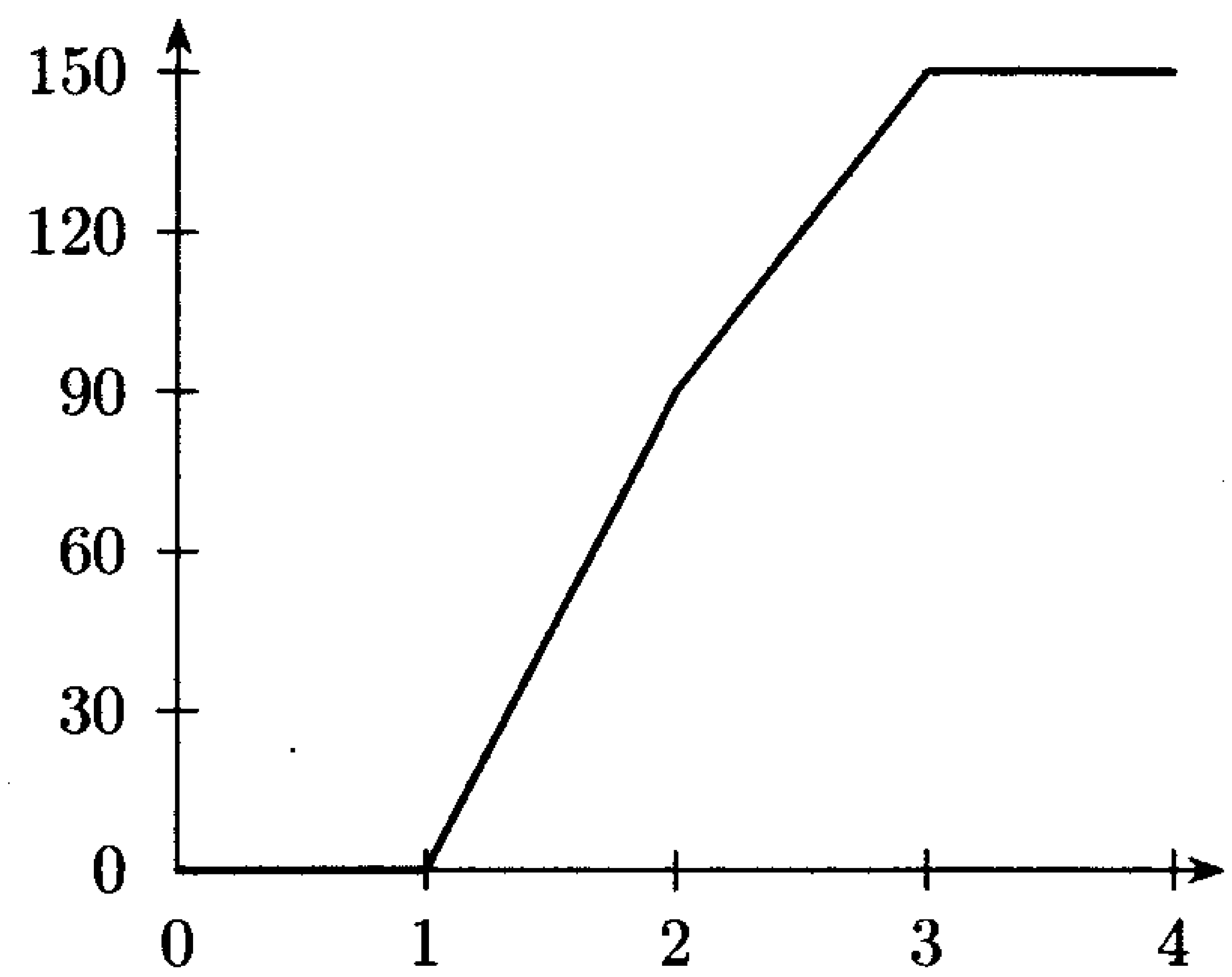
9. (6 points) Find $\int_2^5 f(x)dx$, if $\int_2^5 (3f(x) + 4)dx = 18$.

W08 INT

10. An osprey is flying 400 feet above the sea when it drops a fish it just caught. The fish falls with constant downward acceleration 32 ft/s^2 toward the water.

(a) (3 points) Write an expression for the function $F(t)$ giving the distance (in feet) the fish has fallen t seconds after being dropped.

The osprey begins to dive 1 second after dropping the fish, and its downward velocity is given, in ft/s, by the function $V(t)$, where t is the number of seconds after the osprey dropped the fish. The graph of $V(t)$ is shown below.



(b) (9 points) Write an integral expressing the osprey's average velocity *during its dive*, and evaluate it using the graph.

(c) (3 points) Based on the information given, it is possible for the osprey to recover the fish before it hits the water? Explain.