MATH 115 –SECOND MIDTERM

March 31, 2009

NAME: _____

INSTRUCTOR: _____ SECTION NUMBER: _____

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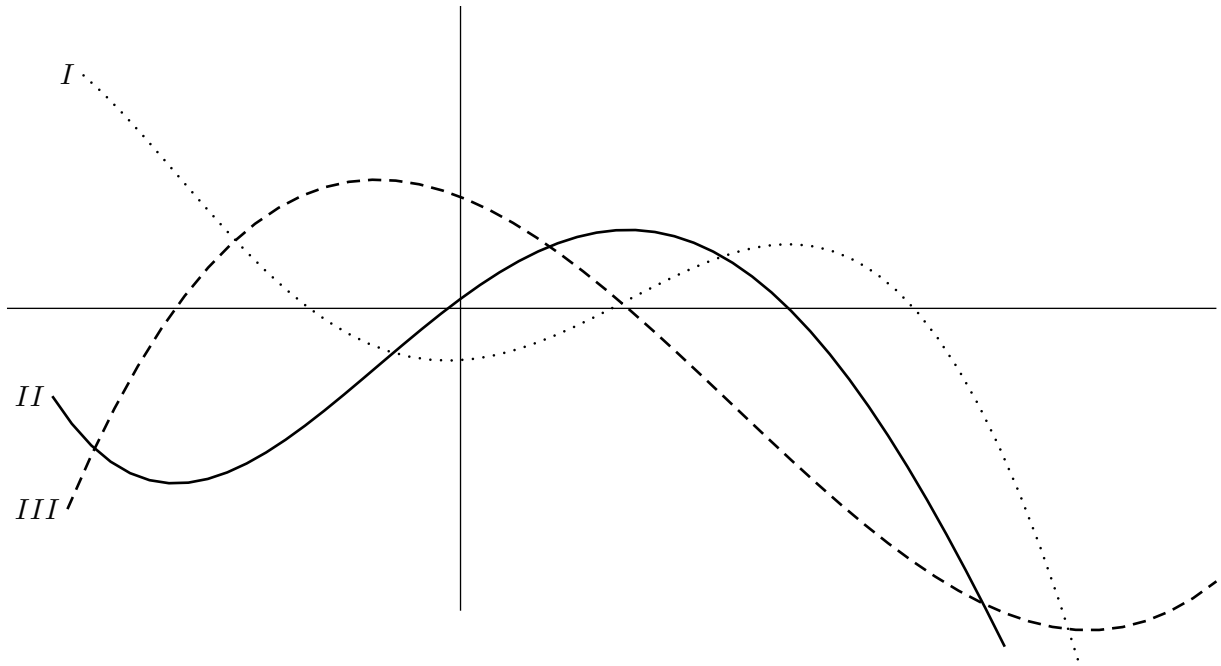
1. **Do not open this exam until you are told to begin.**
2. This exam has 9 pages including this cover. There are 9 questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator. You are also allowed two sides of a 3 by 5 notecard.
7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to show how you arrived at your solution.
8. Please turn off all cell phones and pagers and remove all headphones.

PROBLEM	POINTS	SCORE
1	8	
2	12	
3	16	
4	16	
5	6	
6	6	
7	10	
8	10	
9	16	
TOTAL	100	

1. (8 points) On the axes below are graphed f, f', and f''. Determine which is which, and justify your response with a brief explanation.



I : _____

II : _____

III : _____

Explanation:

2. (12 points) Suppose a is a positive (non-zero) constant, and consider the function

$$f(x) = \frac{1}{3}x^3 - 4a^2x.$$

Determine all maxima and minima of f in the interval $[-3a, 5a]$. For each, specify whether it is global or local.

3. Table 1 below displays some values of an invertible, differentiable function $f(x)$, while Figure 2 depicts the graph of the function $g(x)$. Set $h(x) = f\big(g(x)\big)$ and $j(x) = \dfrac{f(x)}{g(x)}$.

Table 1

x	1	2	3	4	5
$f(x)$	-5	-2	2	4	7
$f'(x)$	5	6	2	3	3
$f''(x)$	1	-1	-3	-2	0

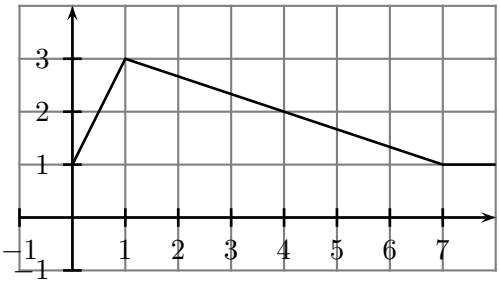


Figure 2: Graph of $g(x)$

Evaluate each of the following. **To receive partial credit you must show your work!**

(a) (4 points) $\big(f^{-1}\big)'(2)$

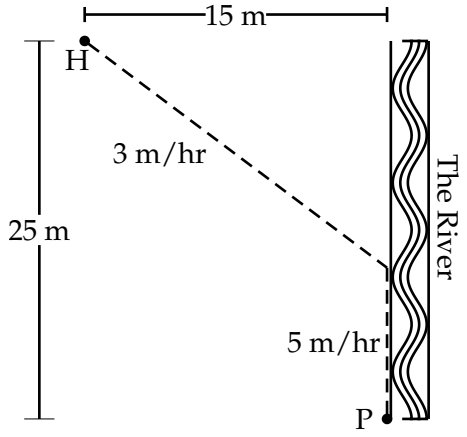
(b) (4 points) $h'(4)$

(c) (4 points) $h''(4)$ [*Hint: you may want to use your work from part (b).*]

(d) (4 points) $j'(4)$

4. (16 points) The Awkward Turtle is going to a dinner party! Unfortunately, he's running quite late, so he wants to take the quickest route. The Awkward Turtle lives in a grassy plain (his home is labeled H in the figure below), where his walking speed is a slow but steady 3 meters per hour. The party is taking place southeast of his home, on the bank of a river (denoted by P in the figure). The river flows south at a constant rate of 5 meters per hour, and once he gets to the river, the Awkward Turtle can jump in and float the rest of the way to the party on his back. A typical path the Awkward Turtle might take from his house to the party is indicated in the figure below by a dashed line.

   What is the shortest amount of time the entire trip (from home to dinner party) can take? [*Recall that rate* $\times$ *time* = *distance.*]



Minimal time = _____

5. Your friend starts a small company which sells awesome t-shirts for \$10 apiece. The table below shows the cost of making different numbers of shirts:

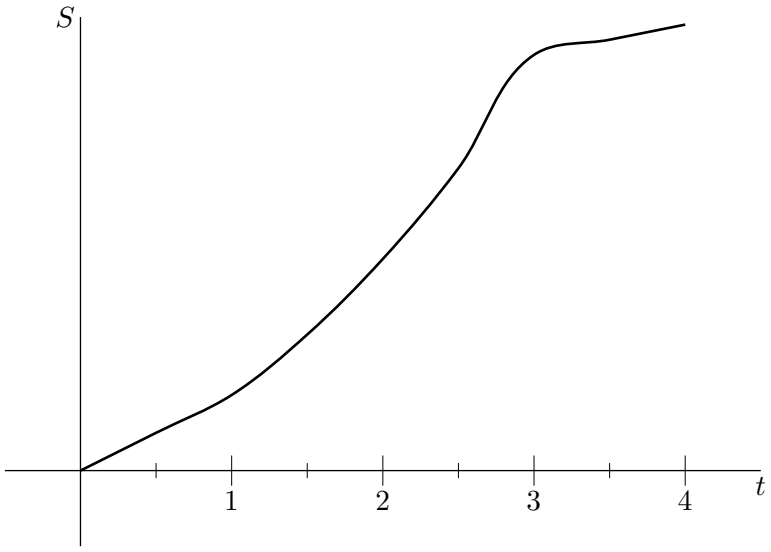
q (number of shirts made)	5	10	15	20	25	30	35	40	45	50
$C(q)$ (cost, in \$)	100	130	150	168	184	196	206	218	236	256

   (a) (2 points) Write an expression for the revenue function $R(q)$.

   (b) (4 points) How many shirts should your friend aim to sell, if her goal is to maximize profit? Explain.

6. (6 points) The radius of a spherical balloon is increasing by 3 cm per second. At what rate is air being blown into the balloon at the moment when the radius is 9 cm? Make sure you include units! [*Hint: the volume of a sphere of radius r is* $\frac{4}{3}\pi r^3$.]

7. You decide to take a weekend off and drive down to Chicago. The graph below represents your distance S from Ann Arbor, measured in miles, t hours after you set out.



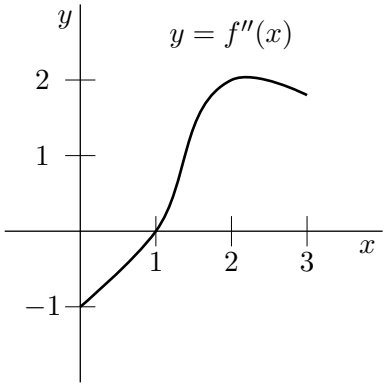
Let $A(t)$ be the slope of the line connecting the origin $\big(0,0\big)$ to the point $\big(t, S(t)\big)$.

(a) (3 points) What does $A(t)$ represent in everyday language?

(b) (3 points) Estimate the time t at which $A(t)$ is maximized. Write a one sentence explanation and use the graph above to justify your estimate.

(c) (4 points) Use calculus to explain why $A(t)$ has a critical point when the line connecting the origin to the point $\big(t, S(t)\big)$ is tangent to the curve $S(t)$.

8. The figure below shows the graph of the *second* derivative of f, on the interval $[0, 3]$.



Assume that $f'(1) = 1$ and $f(1) = 0$.

(a) (5 points) Can $f'(x) = 0.5$ for some x in $[0, 3]$? Why or why not?

(b) (5 points) Explain why f has a global maximum at $x = 3$.

9. (a) (4 points) Suppose that the tangent line to the function $y = f(x)$ at $x = c$ passes through the origin. Express $\left.\frac{dy}{dx}\right|_{x=c}$ in terms of c and $f(c)$.

(b) (6 points) Consider the graph of $xy = ae^{by}$, where both a and b are positive (non-zero) constants. Determine $\frac{dy}{dx}$.

(c) (6 points) Write down the equations of all lines passing through the origin which are tangent to the curve $xy = ae^{by}$, where as before a and b are positive (nonzero) constants. [*Hint: You may find it helpful to rewrite your answer to 9b without exponentials, by using substitution – by the definition of the curve, you can replace the quantity* ae^{by} *by* xy.]