Math 115 – Final Exam

April 23, 2009

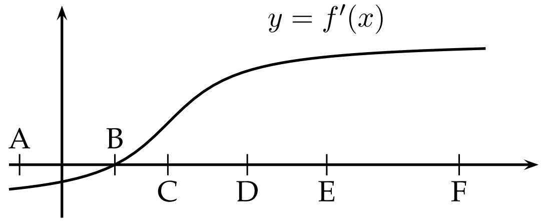
Name: ______________________________

Instructor: ____________________ Section Number: ______________

1. **Do not open this exam until you are told to begin.**
2. This exam has 10 pages including this cover. There are 9 questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator. You are also allowed two sides of a 3 by 5 notecard.
7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to show how you arrived at your solution.
8. Please turn off all cell phones and pagers and remove all headphones.

Problem	Points	Score
1	14	
2	8	
3	9	
4	10	
5	8	
6	14	
7	16	
8	10	
9	11	
Total	100	

1. (2 points each) Suppose f is a twice-differentiable function. Use the graph of the derivative f', shown below, to answer the following questions. No explanations are required.



(a) At which of the marked x-values does f attain a global minimum on the interval [A,F]?

(b) At which of the marked x-values does f attain a global maximum on the interval [A,F]?

(c) At which of the marked x-values does f' attain a global minimum on the interval [A,F]?

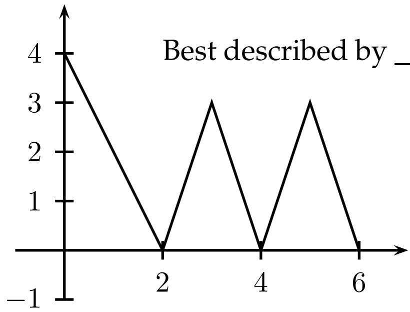
(d) At which of the marked x-values does f' attain a global maximum on the interval [A,F]?

(e) At which of the marked x-values does f'' attain a global maximum on the interval [A,F]?

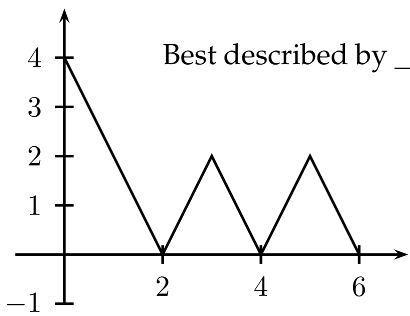
(f) For which of the marked x-values does $\int_A^x f'(t)\,dt$ attain a global minimum on the interval [A,F]?

(g) For which of the marked x-values does $\int_A^x f'(t)\,dt$ attain a global maximum on the interval [A,F]?

2. (2 points each) Next to each of the functions graphed on the left below, identify which one of the inequalities on the right below best describes the situation. Here, L is the left Riemann sum for $\int_0^6 f(x)\,dx$ using three equal subdivisions, and R is the right Riemann sum using three equal subdivisions. [*You may find it helpful to compute L, R, and the integral for each graph.*]



Best described by ____



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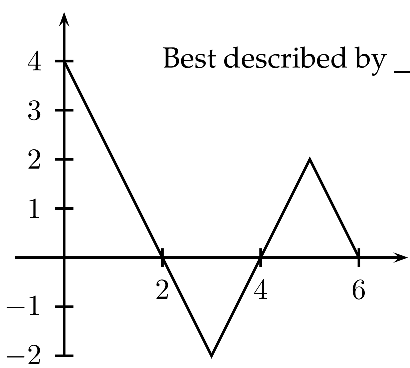

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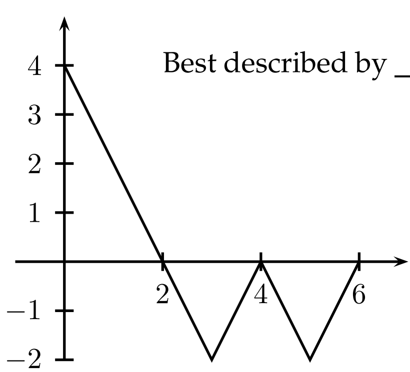


Best described by ____

(a) $L < R < \int_0^6 f(x)\,dx$

(b) $L = R < \int_0^6 f(x)\,dx$

(c) $L < R = \int_0^6 f(x)\,dx$

(d) $L < \int_0^6 f(x)\,dx < R$

(e) $L = \int_0^6 f(x)\,dx < R$

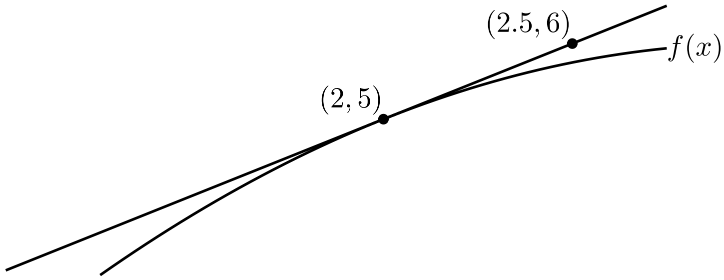
(f) $R < L < \int_0^6 f(x)\,dx$

(g) $R < L = \int_0^6 f(x)\,dx$

(h) $R < \int_0^6 f(x)\,dx < L$

(i) $R = \int_0^6 f(x)\,dx < L$

3. The figure below shows a differentiable function f and the line tangent to the graph at the point $(2,5)$: (*picture not drawn to scale*)



(a) (3 points) Approximate $f(2.01)$. Is your approximation an over or underestimate? Explain.

(b) (3 points) Evaluate $h'(2)$ if $h(x) = \Big(f(x)\Big)^3$.

(c) (3 points) Evaluate $g'(2)$ if $g(x) = e^{f(x)}$.

4. (10 points) A car initially traveling 80 ft / sec brakes to a stop in 8 seconds. Its velocity is recorded every 2 seconds and is given in the following table:

t (seconds)	0	2	4	6	8
$v(t)$ (ft/sec)	80	52	28	10	0

(a) Give a good estimate for the distance the car traveled during the course of the 8 seconds. Is your approximation an over or underestimate? How do you know?

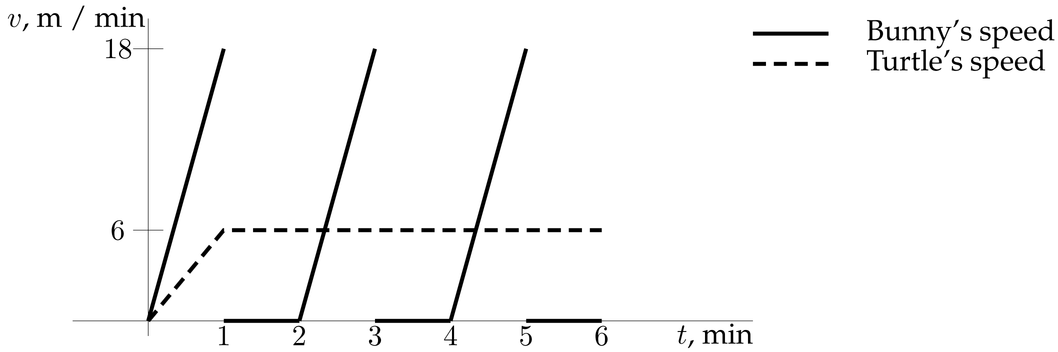
(b) To estimate the distance traveled accurate to within 20 feet, how often should the velocity be recorded?

(c) Approximate the acceleration of the car 4 seconds after the brakes were applied.

5. (8 points) A potter has a fixed volume of clay in the form of a cylinder. As he rolls the clay, the length of the cylinder, L, of increases, while the radius, r, decreases. If the length of the cylinder is increasing at a constant rate of 0.2 cm per second, find the rate at which the radius is changing when the radius is 1.5 cm and the length is 4 cm.

[*Recall that the volume of a cylinder of radius r and length L is $\pi r^2 L$.*]

6. The Awkward Turtle is competing in a race! Unfortunately his archnemesis, the Playful Bunny, is also in the running. The two employ very different approaches: the Awkward Turtle takes the first minute to accelerate to a slow and steady pace which he maintains through the remainder of the race, while the Playful Bunny spends the first minute accelerating to faster and faster speeds until she's exhausted and has to stop and rest for a minute - and then she repeats this process until the race is over. The graph below shows their speeds (in meters per minute), t minutes into the race. (Assume that the pattern shown continues for the duration of the race.)



(a) (6 points) What is the Awkward Turtle's average speed over the first two minutes of the race? What is the Playful Bunny's?

(b) (3 points) The Playful Bunny immediately gets ahead of the Awkward Turtle at the start of the race. How many minutes into the race does the Awkward Turtle catch up to the Playful Bunny for the first time? Justify your answer.

(c) (5 points) If the race is 60 meters total, who wins? Justify your answer.

7. (4 points each) Table 1 below displays some values of an invertible, twice-differentiable function $f(x)$, while Figure 2 depicts the graph of the function $g(x)$.

Table 1

x	1	2	3	4	5
$f(x)$	-5	-2	2	4	7
$f'(x)$	5	6	2	3	3
$f''(x)$	1	-1	-3	-2	0

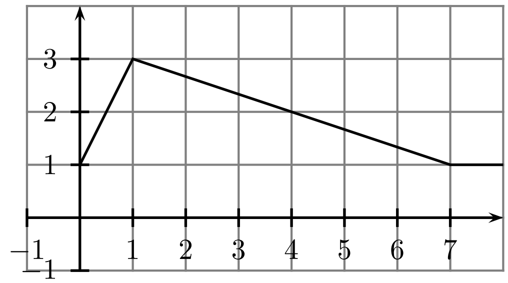


Figure 2: Graph of $g(x)$

Evaluate each of the following. Show your work.

(a) $\displaystyle\int_0^7 g(x)\,dx$

(b) $\displaystyle\int_1^3 f'(x)\,dx$

(c) $\displaystyle\int_1^5 \Big(3f''(x)+4\Big)\,dx$

(d) $\displaystyle\int_1^4 \Big(f'(x)g(x)+f(x)g'(x)\Big)\,dx$

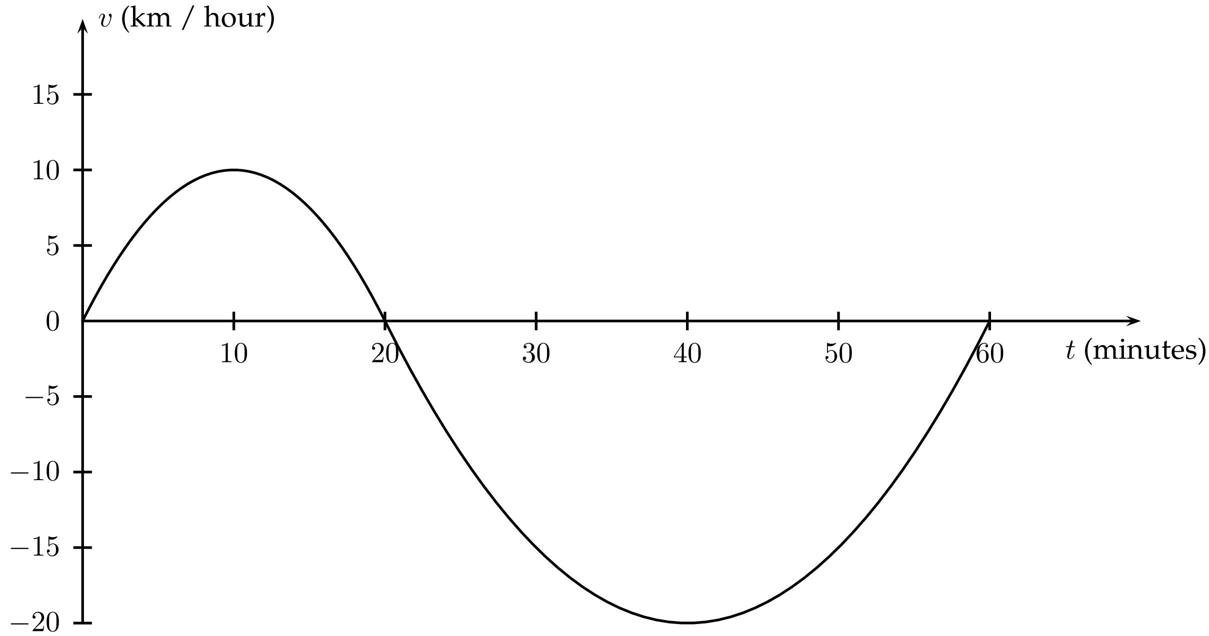
8. (10 points) A typical student spends the 24 hours leading up to this exam sleeping, studying, eating, and Facebook stalking. Suppose the total amount of time spent on eating and Facebook is 8 hours. The student's score on the exam, E (out a possible 100 points) depends on S, the number of hours of sleep the student enjoys during the 24 hours leading up to the exam. To be precise,

$$E(S) = 40 \sin\left(\frac{5\pi}{51}(S - 3.4)\right) + 36$$

How many hours should the student study in the day leading up to the exam to maximize his / her score?

[*You must use calculus - not just your calculator - and show your work to receive full credit.*]

9. A bicyclist is pedaling along a straight road for one hour with a velocity v shown in the figure below. She starts out five kilometers from a lake; positive velocities take her toward the lake.



(a) (2 points) Does the cyclist ever turn around? If so, at what time(s)?

(b) (3 points) When is she going the fastest? How fast is she going then? Is she going toward or away from the lake?

(c) (3 points) When is she closest to the lake? Approximately how close to the lake does she get?

(d) (3 points) When is she farthest from the lake? Approximately how far from the lake is she then?